Introduction to Statistics

Welcome to AP Statistics. For those of you who thought that you were signing up for a math class, I feel that I should make you aware that I really do not consider statistics a math class. I consider statistics a class that uses math much like some of the science classes utilize math. In statistics, like science, we test hypotheses through the design and completion of observational studies and experiments.

Okay so you are asking yourself, if statistics is not really a math course what is it and why is it important?

First, I will address the question: **Why is statistics important?** It is important because I said it is. I am guessing that my answer didn't satisfy many or any of you. Okay, so how about this: How do you think Amazon grew to be so huge and powerful when it started out as online book store? The answer their use of statistics. How do you think that Japan became a major manufacturing power known for high quality automobiles when their country and industry was virtually destroyed by a world war? The answer is statistics. How do you think a baseball team with the lowest salary base became a divisional champion? The answer is statistics. How did a group of MIT nerds get banned from Las Vegas? The answer is statistics. How was an effective HIV screening test created from one that was completely unreliable? The answer is statistics. How are advertisements selected for your computer screen or smartphone? The answer is statistics. How did Target know that a teenage girl that they had never seen was pregnant before her father did? The answer is statistics.

Let's explore the story about Target and the teenage pregnancy in a little more detail.

The background: To make better use of their advertising budget, Target began utilizing statistics to better identify the purchasing patterns of their customers so that they could send advertisements that might be of greater interest to each customer. In other words, based on what a person purchased, Target was sending advertisements and coupons to them for things that Target was predicting that the customer would want to purchase.

To that end a teenage girl was receiving advertisements for baby items. As a result an angry father walked into a Target outside of Minneapolis demanding to talk to the manager: "My daughter got this in the mail!" he said. "She's still in high school, and you're sending her coupons for baby clothes and cribs? Are you trying to encourage her to get pregnant?"

The manager didn't have any idea what the man was talking about. He looked at the mailer. Sure enough, it was addressed to the man's daughter and contained advertisements for maternity clothing, nursery furniture and pictures of smiling infants. The manager apologized and then called a few days later to apologize again. On the phone, though, the father was somewhat abashed. "I had a talk with my daughter," he said. "It turns out there's been some activities in my house I haven't been completely aware of. She's due in August.

The Complete article can be found: **http://www.forbes.com/sites/kashmirhill/2012/02/16/how-target-figured-out-a-teen-girl-was-pregnant-before-her-father-did/**

A closely related article and interview of the Target advertising Mastermind: **https://www.nytimes.com/2012/02/19/magazine/shopping-habits.html?pagewanted=6&_r=1&hp**

So now that you are a little curious, let's answer the question: **What is Statistics?**

Statistics is the study of how to collect, organize, display, describe, and analyze the data from a sample in order to make generalizations about a population.

I am certain that some or many of you are thinking, great another definition that my teacher believes explains everything, but in reality it leaves more questions than answers. To which I must respond, fair enough, but give me a few more minutes and I believe that I will be able to break down the definition and give you an overview of the course at the same time.

AP Statistics is divided into 4 sections:

- Collecting Data,
- Displaying and Describing the Collected data
- Analyzing the Collected Data
- Making inferences/decision based on the data collected.

The First Quarter will be devoted to Collecting Data and Displaying and Describing data. These two sections are the easiest sections of the course and it is imperative that you achieve really good grades in both of these sections for two major reasons. First, the collection of data is foundational to all other sections of the course and secondly the 2nd quarter which is comprised almost entirely of data analysis, **(the probability section)** is the most challenging for students. Yes, most students say probability is the hardest and their second quarter grades tend to be lower than that of their first quarter.
The final section of the course, making inferences, is where you actually get to make decisions based on data. Should we purchase the new machine, is the new drug more effective than the old one, can blank predict blank and with how much confidence.

I love the section on inferences and I feel it is empowering, because it is from this section that I can use all of the collected data to make an informed decision and not just a guess. Now I want to be perfectly clear about the word guess. **Nothing in statistics is certain,** however, with statistics I am able to determine the chance that I could be wrong. Typically I will choose to be correct 95% of the time. Which means that I accept the probability that I will be wrong about 5% of the time—but I am getting ahead of myself as these discussions will not occur in detail until the 3rd quarter.

As we begin this course, I should warn you that there are a significant number of definitions that must be learned in order to understand what we are trying to accomplish. So how important are the definitions? Very important—put it this way, I don't expect you to have to perform any mathematical calculations on your first test. That isn't to say that the first test is easy, it is just a recognition that statistics is different from all of the math classes you have taken thus far.

Incidentally, the first test is challenging for most students, not because the topics are difficult but because you are being asked to think and reason in a different manner. In previous math classes, you followed an explicit set of directions to find a solution to a problem. In statistics, you will be expected to not only know and apply the definitions and methods, but be able to relate them to the outside world. In fact, the more you know about the world and other subjects, the more connections you will be able to make and the more successful you will be in this course.

So back to the definition and why we need statistics:
Statistics is the study of how to collect, organize, display, describe, and analyze the data from a sample in order to make generalizations about a population.

To fully understand what this means, we need to understand the concept and importance of both a **population** and a **sample**. (we will define and discuss **sample** in detail next week)

Population: the entire group of subjects or individuals that is the subject of interest.

Parameter-a numerical measurement of a population *(rarely known).*

Census-the collection of data from each unit in the population.
(*Difficult if not impossible with a large population*)

In statistics, a population is the entire group of subjects or individuals that is the subject of interest while a sample is a subset of the population. For example: we may be interested in the percentage of students in your statistics class that have brown eyes. In that case, our population would be the students in your statistics class and it would be fairly easy to count and figure out the percentage of brown eyed students in the class. The percentage of brown-eyed students would be the parameter. Because we are checking the eye-color of the entire population, every student in the classroom, we are conducting a census.

The great thing about being able to conduct a census, is that we know the true population parameter. In this case, we would know the true proportion or percentage of students with brown eyes in the classroom. According to some of my students, the best part of being able to conduct a census knowing the true population parameter is that there is no need for statistical methods and no need for this class.

So why not perform a census all of the time? To that answer that question let's consider the percentage of students with brown-eye color scenario. What if my population of interest is not the students in my classroom but all of the students at Reagan? While counting that many students might be a challenge, and I might creep a few people out when I asked them for their eye color, it is probably still doable. However what if we wanted to know the percentage of people in San Antonio or the world who had brown eyes? Well then our population would be all of San Antonio or the world and it would be unrealistic to conduct a census count all of the brown eyed people in San Antonio or the world.

Why don't we always collect data using a census?

1. Sometimes it is impossible to conduct a census.
    - I cannot find the average size of bass in a particular lake without draining the lake at which point the bass is dead and there is no lake.
    - I cannot count the number of red blood cells in your body, without removing all of the blood from you and sucking your life away.
2. A census can be extremely expensive
   The 2010 U.S census cost **$13 billion**, **approximately $42** person—your tax dollars at work.
3. Due to the difficulty in execution, a census can very inaccuarate
    - In the 2000 census, it is estimated that 6.4 million Americans were missed, however, there was an overcount of 36,000. Now how did that happen?
    - In 2010, it is estimated that in Hildalgo county in Texas 225,000 persons were not counted.

Notes: Simulations

So while an accurate census is great, it is not always possible and may be cost prohibitive. In the event that it is not plausible to conduct a census, we are forced to settle for an estimate. So the questions is how do we make an accurate estimate. Essentially we have two choices, we can take a sample or we can run a simulation. We will discuss in great detail how to utilize sample data, but we are going to begin with learning how to conduct a simulation.

Simulation: a way to model random events, such that simulated outcomes closely match real-world outcomes. By observing simulated outcomes, researchers gain insight on the real world.

- Chance/randomness must be employed
- Often Used because it is more economical than running a true experiment
- Typically a probability model can be used to generate the same information
- We Want things to be fair and without bias or prejudice in our method of selection

Random: An event in which we know what possible outcomes can occur but do not which outcome actually will take place.

Component: The most basic **situation** in which something happens at random—(Hint: the singular event that you are repeating)

Outcome: the result of a single component

Trial: The number of components necessary to occur to model a situation. A single run of sequence of events being simulated

Response Variable: variable that measures the outcome of each trial; **outputs (Y's)**

Example: How many heads would I get if I flipped a coin 100 times?

Modeling a Simulation

Step 1: Identify the component to be repeated (a component is the most basic event of the simulation

Step 2: Explain how to model the outcomes (Assign numbers to the possible outcomes)

Step 3: Explain how to simulate the trial-what are you going to do

Step 4: Clearly State what the Response Variable is—(this is the answer to the question)
Hint: How many random selections did it take to complete a trial?

Step 5: Run Several Trials (chart the results)
Remember: A trial is the desired outcome

Step 6: Analyze the Response Variable (take an average)

Step 7: State your conclusion in the context of the problem.

Recipe for Success: Simulations

1. Read the entire problem

What is being asked?

2. Identify the Question

Explain what the question is asking in your own words

3. Identify and define a success and component in context

- **Success**: What we want to happen
- **Component:** What is being repeated

4. Identify a Trial

How many successes are required?

5. Model the simulation

Assign 1 or 2 digit numbers in proportion to the chance of success and failure.

- 00 = 0 or 100; 01=1; 02 = 2; 03 = 3…09 = 9
- 10 = 10; 11 = 11… 99 = 99
- **Assign the numbers to be skipped or ignored**

6. Address Duplications

Are repeats permitted: Can something occur twice?

- Percentages-usually can be duplicated
- Specific items-usually cannot be duplicated (Occurs when the quantities of items are known)

7. Explain how to run the simulation

1. **Explain how to run a trial**
   - Beginning from left to right I would select 1 or 2 digit numbers until there were ___ number of successes.
   - Count how many 1 or 2 digit values that were not skipped.
2. **Tell how many trials are going to be run**
3. **Find the average/mean of all the trials**

8. Run the simulation & Make a Table

Trial Number	Number of 2 Digit Values Counted (successes)
1	
2	
3	
	Total

- Draw a line through the values that represent failures
- Circle the values that represent successes (Do not forget about duplicates are they permitted or not)
- Mark an X through Skips (These are numbers that are not possible- for instance when duplicates are not permitted)
- Draw a vertical line at the end of a trial
- Count the number of 2 digit numbers in the trial
- Record the values in a table
- Repeat for all necessary trials to complete the simulation

9. Calculate the Expected number

Take the average/Mean

Sum the number of successes counted for each trial

- Divide by the number of trials

Conclusion:

Based on the simulation above, on average we would expect to have

_____ in order to find_____

Give the number & definition of component *Give the number & definition of successes*

Notes: Simulations Scenarios

Scenario 1: Every Monday a local radio station gives coupons away to 50 people who correctly answer a question about a news fact from the previous day's newspaper. The coupons given away are numbered from 1 to 50, with the first person receiving coupon 1, the second person receiving coupon 2, and so on, until all 50 coupons are given away. On the following Saturday, the radio station randomly draws numbers from 1 to 50 and awards cash prizes to the holders of the coupons with these numbers. Numbers continue to be drawn without replacement until the total amount awarded first equals or exceeds $300. If selected, coupons 1 through 5 each have a cash value of $200, coupons 6 through 20 each have a cash value of $100, and coupons 21 through 50 each have a cash value of $50.

(a) Explain how you would conduct a simulation using the random number table provided below to estimate the distribution of the number of prizewinners each week.

(b) Perform your simulation 3 times. (That is run 3 trials of your simulation.) Start at the leftmost digit in the first row of the table and move across. Make your procedure clear so that someone can follow what you did. You must do this by marking directly on or above the table. Report the number of winners in each of your 3 trials.

72749	13347	65030	26128	49067	02904	49953	74674	94617	73317
81638	36566	42709	33717	59943	12027	46547	61303	46690	76423
38449	46438	91579	01907	72146	05764	22400	94490	49890	09258

Recipe for Success: Simulations

1. Read the entire problem

What is being asked?

2. Identify the Question

Explain what the question is asking in your own words

3. Identify and define a success and component in context

- **Success**: What we want to happen
- **Component:** What is being repeated

4. Identify a Trial

How many successes are required?

5. Model the simulation

Assign 1 or 2 digit numbers in proportion to the chance of success and failure.

- 00 = 0 or 100; 01=1; 02 = 2; 03 = 3…09 = 9
- 10 = 10; 11 = 11… 99 = 99
- **Assign the numbers to be skipped or ignored**

6. Address Duplications

Are repeats permitted: Can something occur twice?

- Percentages-usually can be duplicated
- Specific items-usually cannot be duplicated
  (Occurs when the quantities of items are known)

7. Explain how to run the simulation

4. Explain how to run a trial

- Beginning from left to right I would select 1 or 2 digit numbers until there were ___ number of successes.
- Count how many 1 or 2 digit values that were not skipped.

5. Tell how many trials are going to be run

6. Find the average/mean of all the trials

8. Run the simulation & Make a Table

Trial Number	Number of 2 Digit Values Counted (successes)
1	
2	
3	
	Total

- Draw a line through the values that represent failures
- Circle the values that represent successes
  (Do not forget about duplicates are they permitted or not)
- Mark an X through Skips
  (These are numbers that are not possible-
  for instance when duplicates are not permitted)
- Draw a vertical line at the end of a trial
- Count the number of 2 digit numbers in the trial
- Record the values in a table
- Repeat for all necessary trials to complete the simulation

9. Calculate the Expected number

Take the average/Mean

Sum the number of successes counted for each trial

- Divide by the number of trials

Conclusion:

Based on the simulation above, on average we would expect to have

_____ in order to find_____

Give the number & definition of component *Give the number & definition of successes*

Notes: Simulations Scenarios

Scenario 2: Suppose the probability that an exploratory oil well will strike oil is about 0.2. Conduct a simulation to answer the following questions. Assume that the outcome (oil or no oil) for any one exploratory well is independent of outcomes from other wells. Use the random number table below and conduct 20 trials. Clearly identify each trial on the table.

(a) Describe the simulation and estimate the average number of wells that need to be drilled in order to strike oil.

(b) What is the probability that it will take fewer than 3 attempts to strike oil?

(c) What is the probability that it will take exactly 6 wells to strike oil?

47169	80410	03333	73856	85627	54351
36653	55390	20439	48605	45513	05458
76361	47409	14914	55280	70533	52960
20579	87054	59998	90071	67554	91237
96994	65965	73235	49260	45309	24660
92048	08676	72653	87342	19084	33780
37592	96361	18246	36121	14888	23329
08032	20831	98314	93521	24035	43186

Notes: Simulations Scenarios

Scenario 3: A certain professor has eight keys, but he never recalls which one fits his office door lock. He places all of the keys on a table and randomly selects and tries one key at a time. If the key does not fit he places the key back in the original pile and then randomly selects another key. *(yes, he is senile and probably teaches statistics-don't laugh that would be rude*) **Note: all the keys look the same and he is unable to tell which key he has already tried.** Conduct a simulation to answer the following questions. Use the random number table below and conduct 20 trials. Clearly identify each trial on the table.

(a) Explain how you would conduct a simulation using the random number table provided below to estimate the number of keys the professor would need to try in the lock.

(b) What is the expected number of tries needed for him to find the correct key?

(c) What is the probability it will take more than 4 tries to find the right key?

64831	78558	25961	07610	75464	85326
34336	39840	24371	53548	01485	57845
11792	38659	92620	48253	05370	80411
65985	43392	21100	08763	37469	66583
52822	48990	03648	34861	54680	64791
31645	45552	78255	64794	21228	69707
38804	45687	85320	54654	76156	01853
97115	91205	92396	97645	18911	76701
29815	81029	06361	86378	33931	09331

Notes: Samples

While economical, easy to run, and unbiased, simulations have 2 major limitations.

1. **To run a simulation we must know something about the population**. We must know the population parameter of interest. For instance, in the oil well simulation, we needed to know the percentage of time that we expected to strike oil. Unfortunately, much of the time we often don't know anything about the population. We don't know the percent of deformed blood cells in your body; we don't know what percent of the population supports a candidate.

2. **Simulations do not allow us to test whether or not the claimed percentage of successes is correct.** When running a simulation we are forced to assume that the percentage of successes is correct. The oil well simulation problem claimed that we struck oil twenty percent of the time, but what if the percentage had been different than 20 percent and we the drilling company were unaware of the true percentage of successful wells? We assumed that the drilling company would strike oil twenty percent of the time and used that value in our simulation. However, simulations do not provide a mechanism to determine whether or not this was actually the case.

So the question is: what do we do when we don't know a population parameter and conducting a census is not appropriate? **The answer is:** take a sample. Well that sounds simple enough, but leads us to the next question: **What is a sample?** A sample is merely a subset of the population.

Scenario: If I wanted suggestions for a theme song for this year's Reagan students, instead of performing a census and asking all of the students at Reagan for their suggestions, I could take a sample or subset of Reagan students. Since a sample is a subset of the population there are several ways to go about finding a subset.

1. I could ask twenty-five students sitting together in the lunch room.
2. I could ask my freshmen advisory and ask for their idea of the theme song.
3. I could go into the Teacher's lounge and get suggestions for the song there.
4. I could attend the PTA meeting and ask for their suggestions.

So which of those methods would you suggest I use and how do you feel about them?
I am going to go out on a limb here and guess that you don't really like any of those methods. I am betting that you are concerned that your opinion for the Reagan Theme Song may not be represented.

While samples can be an excellent way to make determinations about a population, **samples must be representative of the entire population**. So the question is: how do we take a "good sample."

First, I want to stress again that **a good sample is one that is representative of the population**. While that may seem obvious, it isn't as easy to do as one may think. If you are having trouble believing me, consider who I should ask about choosing a theme song for Reagan and give me a list of those people.

Summary:

- A sample is a portion of a population which is studied to draw conclusions & about the characteristics of the whole population.
- A sample must be representative of the population to be useful.
- No conclusions should be drawn from poorly collected data or badly designed experiments or studies.

Notes: Samples and Bias

We have touched on the need for "good samples," samples that are representative of the population of interest. We have also suggested that collecting a representative sample is not always easy to do. *(Did we ever get an agreement as to whom we should talk to about a theme song for the school).* So what are some of the pitfalls that prevent us from collecting samples that are representative of the population?

As preparation for those answers, I would like to ask you all to participate in a brief but very important activity. Without giving it much thought, please select number and circle it.

1 2 3 4

How many of you chose: 1. _____ 2. _____ 3. _____ 4._____

Typically about 75% of the population chooses 3, 20% are divided between 2 and 4 and 5% choose 1. Because there are 4 numbers, each number should be chosen 25% of the time. So why didn't that happen. Psychologists have discussed a variety reasons why we overwhelmingly choose 3 and fail to choose 1. However regardless of the reason, this shows that in general we make biased selections and in statistics biased samples result in samples that are not representative of the population and are therefore worthless.

So what is bias? Bias is over or under-representing a component of the population in a sample.

The above example has some really practical applications to your life. You don't thinks so? Consider this: You go to the bathroom and there are three stalls. Assuming the bathroom is empty, which stall do you normally choose? For the vast majority of people, the answer is the second stall or middle stall which makes it the grossest and least clean stall of all. The next most chosen stall is the third stall which is the furthest one from the door. Assuming we are concerned about germs, hygiene etc. we should be choosing the first stall. Who knew that stats class would be so helpful with toilet training?

In the scenarios above we exhibited a bias. We over-represented the number 3 and the 2nd stall and we significantly under-represented the number 1 and the 1st stall. To have samples that are useful, we must eliminate bias through the design and execution of our studies and experiments.

Summary: The key point is that we must design studies in a manner that selects samples that are representative of the population. We must not over or under-represent segments of the population.

Notes: Bias

Bias is the result of a bad study design and is sometimes referred to as **selection bias**, because the sample we **selected is biased,** that is to say that the selected sample does not represent the population.

As we stated earlier, biased data is absolutely useless and we have no methods to fix data that is not representative of the population of interest. If we happen to collect a biased sample, then we must discard it, start over and collect another sample.

In order to avoid collecting a biased sample, it is important to recognize the types of bias that exist so that we can create data collection methods that prevent the introduction of biased data into our sample.

Common types of statistical bias are:

Under-coverage Bias-Occurs when members of a population are not able to be selected or chosen for a sample because they are not able to be accounted for *(homeless, inmates, no land-line, college students).* It is desirable to have a listing of the population and then randomly select individuals from the listing for the sample. Obviously, the sample is only as good as the listing of the population and in this case the above mentioned groups are often left off of population lists.

Non-response Bias-Occurs when a member in the population is randomly identified and selected but refuses to participate in the poll or chooses not to return the survey or they were called but chose not to answer the questions in the poll. In the non-response bias situation, the sample group does mirror the population, but a person or persons in the sample group refused to participate.

Voluntary Response Bias occurs when anyone is permitted to choose to respond to a general invitation such is the case with radio call-in surveys; write in surveys; internet polls. The problem with this type of data collection is that it tends to over-represent people with strong opinions because they are the only ones who care enough to answer the poll. The person in the survey chooses to participate rather than be randomly selected from a listing of the entire population. There is no method to choosing the sample group. The respondents choose themselves.

Response Bias results from questions that are embarrassing/sensitive or are Non-neutral or poorly worded questions which due to their phrasing tend to lead people to a particular response. We minimize the chance of response bias by carefully wording and field testing questions. If a question is sensitive or embarrassing the respondent may choose to lie. Poorly worded questions tend to influence the responses of individuals.

Example of a sensitive question where someone might be prone to lie:
Coach Ham asks the team if anyone one of them had stayed out partying that weekend. Of course, none of the athletes were.

Examples where the wording of the questions influenced the response:
Are you in favor of providing school lunches for children that do not have enough to eat so that they have a chance to have a normal life?

Are you in favor of the government raising your taxes and taking your money away to pay for food for people who don't work? Obviously, the two questions will result in different responses.

Notes: Bias Scenarios

Scenario 1: Interested in determining the percent of the population who believed in God, a surveyor stood outside a church on Sunday morning and asked all of the congregates a neutrally worded question about whether or not they believe in God. Will this survey produce biased results? Explain.

2008 Question 2: A local school board plans to conduct a survey of parents' opinions about year-round schooling in elementary schools. The school board contacts 500 of the families by mail. The survey question is provided below.

> ***A proposal has been submitted that would require students in elementary schools to attend school on a year-round basis. Do you support this proposal? (Yes or No)***

The school board received responses from 98 of the families, with 76 of the responses indicating support for year-round schools. Based on this outcome, the local school board concludes that most of the families with at least one child in elementary school prefer year-round schooling.

a. What type of bias is included in this survey?
b. Could we just contact another 500 families and add their data to the original results?
c. Name 2 ways to address the bias in this survey?

Notes: Bias Scenarios

2004 Form B Question 2: At a certain university, students who live in the dormitories eat at a common dining hall. Recently, some students have been complaining about the quality of the food served there. The dining hall manager decided to do a survey to estimate the proportion of students living in the dormitories who think that the quality of the food should be improved. One evening, the manager asked the first 100 students entering the dining hall to answer the following question.

> Many students believe that the food served in the dining hall needs improvement. Do you think that the quality of food served here needs improvement, even though that would increase the cost of the meal plan?
>
> _____ Yes _____ No _____ No opinion

(a) In this setting, explain how bias may have been introduced based on the way this convenience sample was selected.

(b) In this setting, explain how bias may have been introduced based on the way the question was worded

(c) How could the question have been worded differently to avoid that bias?

Notes: Sampling Error

Recall that we defined **statistics** as the study of how to collect, organize, display, describe, and analyze the data from a sample in order to make generalizations about a population parameter of interest. Also remember that the parameter is a numerical measurement of a population which could be the mean, median, variance, range, percent…etc. From the sample, or subset of the population, we find estimates of population parameters. The **estimate of a population parameter that comes from a sample is known as a sample statistic and is often referred to as a statistic**. Thus, we talk about the sample mean, or sample median or sample variance…etc. being estimates or approximations for the corresponding population parameters.

For instance, if I wanted to know the true number of hours that Reagan students sleep on average per week, I could find an estimate by taking a sample 50 Reagan students. If I took a "good sample" one that represents the population at Reagan, my findings should be pretty close to the actual number of hours that are slept per night. However, the value I calculated from my sample is likely to differ somewhat from the actual number of hours slept. The difference between my sample statistic and the population parameter is known as sampling error or sampling variation.

More specifically, **Sampling Error or Sampling Variation** is variation due to random chance and can be described by a probability model and can be minimized by increasing the size of the sample. Sampling error is the recognition that every randomly selected sample from a population is likely to be different. Because there are so many variables or factors within a population, it is nearly impossible to design studies that account for every variable without drawing a census, which in itself is difficult, if not impossible. Because we cannot account for all of the variables, we involve chance in the selection of our sample to equalize the effects of variables that could not be accounted for. Even with a well-designed study or experiment, **the sample statistic is only an estimate of the population parameter**. The difference between the two is known as sampling error.

Recall that when we ran our simulations, there were multiple approaches that were all considered acceptable and led to different responses. However as we increased the sample size, the amount of variation in our responses decreased. It makes sense that if we increase our sample size that our result would be closer to the true population parameter. We can say that as we increase our sample size that the difference between the population parameter and the sample statistic will be less and that leads us to say that **the amount of variation diminishes as our sample size increases.** Obviously, our observations may be more varied as the sample size increases, but on average the sample statistics will vary less.

Caution: Please **do not confuse sampling error with Bias**. Sampling error is a term used to describe properly collected data that didn't actually match the population. In other words, given a proper method of data collection was employed, I would expect that my sample will vary or differ from the population the amount of variation or difference of a sample from the population is error. Error is normal and is predictable and therefore can be controlled. **The best way to minimize sampling error/sampling variation is by increasing the sampling size.**

Unlike sampling error, bias is problem in the design and/or method of collecting data. Increasing sample size will not fix biased data. Biased data cannot be fixed and must be discarded. To eliminate bias, we must carefully plan and execute how we collect our data.

Notes: Observational Studies & Randomness

Obviously, it is important to collect samples that are representative of the population and by now you should be asking yourself: **"how do we do that?"** One primary method used to collect samples is the Observational Study. Observational studies are studies in which we gather data about subjects doing what they are already doing. The research of historical records or medical records and surveys are examples of observational studies. Most data is collected through observational studies. Observational studies do NOT assign treatments. **Because treatments are not assigned, causation cannot be proved.** However, observational studies can be statistically significant and can demonstrate a relationship, association or a correlation.

Note: Only Experiments should be used to establish cause and effect relationships.
Most data is collected by observational study and not by controlled experiments.

A **common type of observational study is the survey** of which there are several types including:

- The Census
- The Convenience Sample
- The Simple Random Sample
- The Systematic Sample
- The Cluster Sample
- The Stratified Sample

The Census: As we discussed earlier, a census involves the collection of data from each unit in the population. Unfortunately, a census is often impractical if not impossible to perform. On the positive side, if we perform an accurate census, we will have the population parameter of interest and inference procedures will not be necessary.

The Convenience Sample: The convenience sample is a sampling technique which selects subjects because of their convenient accessibility and proximity to the researcher. Convenience samples are rarely representative of the population and not every subject in the population has an equal chance of being selected. As a consequence, convenience sample findings are most often of no value, however, they are easy to perform if you are interested in collecting worthless data. In other words, **avoid them.**

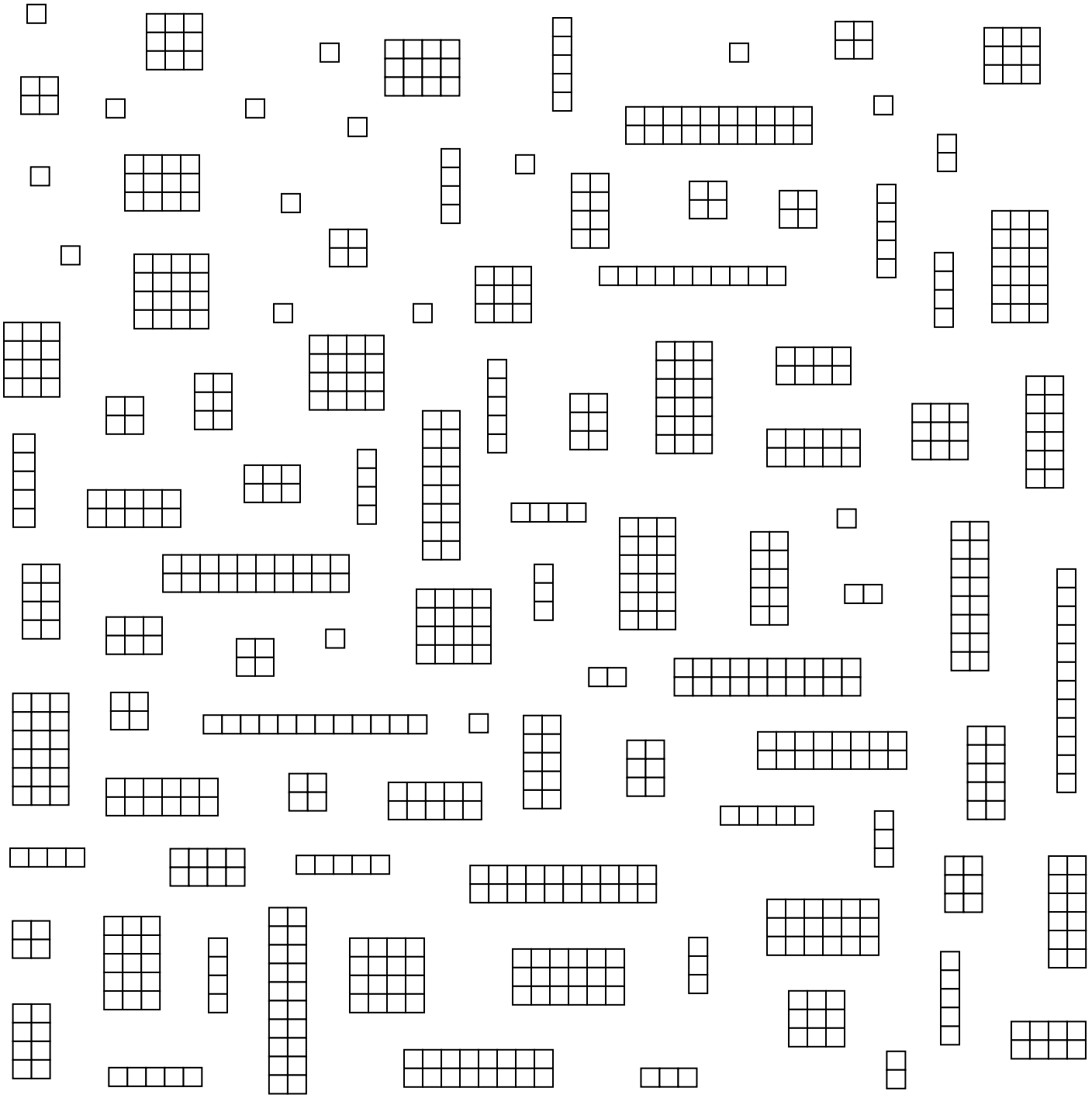
The purpose of Randomness.
Recall that we want our samples to be representative of the population and that means we would desire that we would like every combination of traits to be proportionally represented in our sample. Unfortunately, if we really break it down, we are all unique individuals. Thus if we were to represent every combination of traits, we would need to perform a census and sample everyone. As we discussed previously, this is impractical if not impossible.

So how do we address the problem? The answer is the introduction of chance or randomness. Statisticians and mathematicians have found that randomness, tends to equalize the variations that we can't account for in our studies.

So you don't believe me. Try the exercise on the next page.

Notes: Sample Surveys & Random Rectangles

So let's begin with your best guess as to the mean (average) number of squares in each figure? _____
Please use whole numbers.



Judgment Sample – Select a sample of 5 rectangles that, in your judgment, re representative of the rectangles. Record the size of your five rectangles and find their mean.

_____ _____ _____ _____ _____ mean = _____

Number each figure 1 to 100. The 100 figures make up our population of rectangles.

Notes: Sample Surveys & Random Rectangles

So how do we determine the average number of rectangles per figure? Obviously, we could conduct a census and count the squares in each figure, and take an average. However, I am guessing that many of you would refuse to do the assignment and I am guessing that among those that did the assignment we would still have varying answers. So instead of conducting a census or just using our best guess or our judgment, let's utilize some sampling techniques the employ randomness. We are going to use samples of size 5 which are pretty small. We will begin with the simple random sample.

The Simple Random Sample: This is the basis for all statistical sampling and differs from census and the convenience sample in that randomness is utilized. In a simple random sample all subjects have an equal chance of being selected and every possible combinations of subjects has an equal chance of being selected. That sentence is so important, that I am going to say it again: **In a simple random sample all subjects have an equal chance of being selected and every possible combinations of subjects has an equal chance of being selected.**

So how do we conduct a Simple Random Sample? Typically a number is assigned to all subjects in the population and then a sample of numbers is randomly selected. Acceptable methods of selecting the numbers include: random number generators; mixed numbers in a hat; random number tables. If a **random number table** is used, it is necessary to describe the **selection process**, provide the **stopping point** and define how **duplicate values** are to be addressed. Unfortunately, it is often difficult to obtain a list of the entire population and there is a chance that our sample will not be representative of the population.

Use the random number generator on your calculator to select 5 numbers for our sample.

Calculator Commands:

Press **MATH** → Highlight **PROB**
↓Highlight **8: randIntNoRep(**
Press **ENTER**
Lower: 1 Upper: 100 n: 5 Press **ENTER 2x's**

Locate the corresponding rectangles and record their size and find the mean.

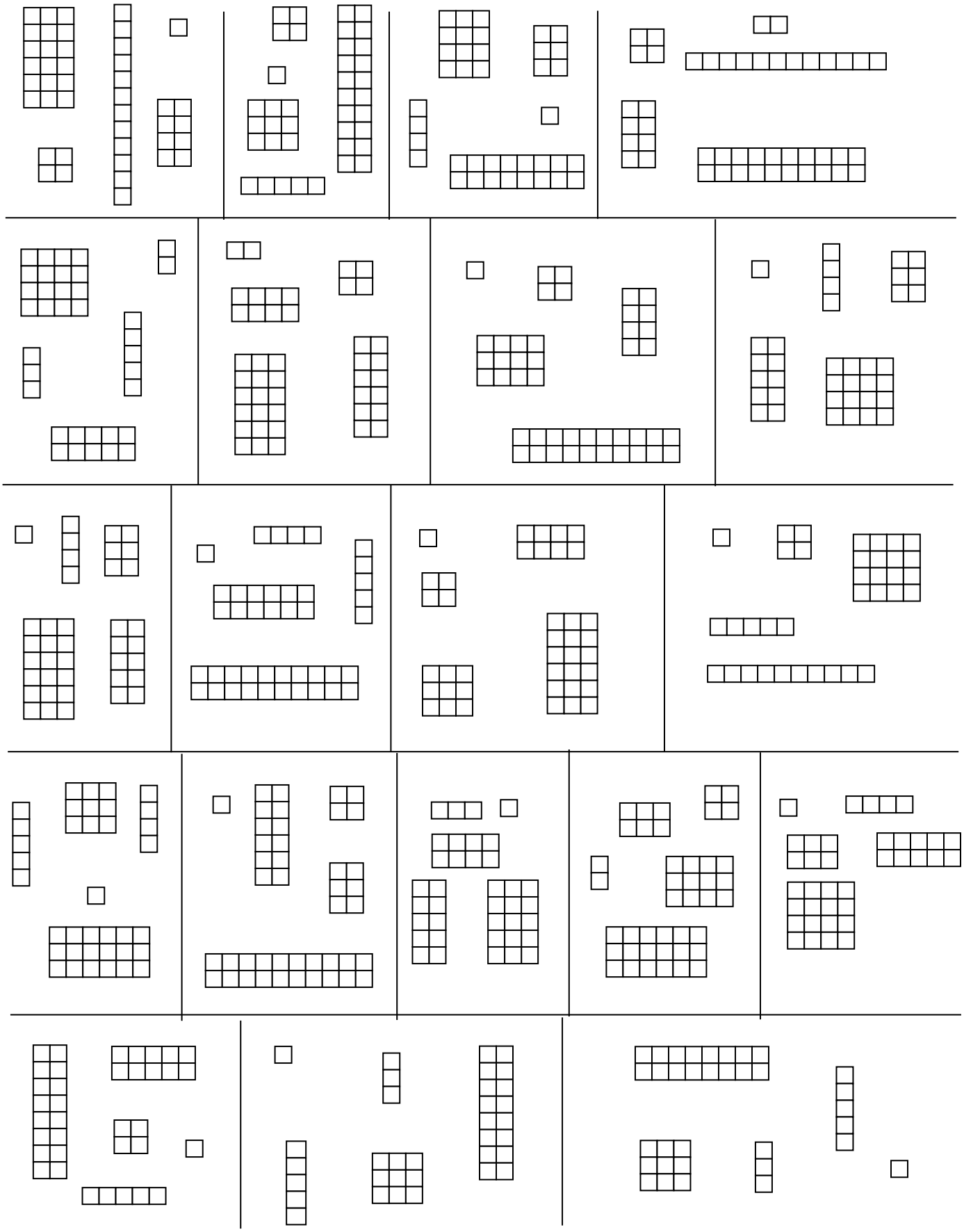
_____ _____ _____ _____ _____ mean = _____

The Systematic Random Sample: The systematic sample also involves randomness and every individual has an equal opportunity of being selected, however ever group is not possible. To perform a systematic random sample begin by dividing the population by your desired sample size which gives us n. We will be selecting every nth value. Systematic sampling is fast and easy and is a good method if there is no pattern in the population. We often use it when we want to survey people coming into a building (selecting every nth person) or every nth thing in a list.

In our case, we will divide 100 by 5 and get 20. Thus we will be selecting every 20^{th} figure in the list. We will also use the n to find our starting point. We do this by using the random number generator and selecting one value 1 to 20. Please perform a systematic random sample.

_____ _____ _____ _____ _____ mean = _____

Notes: Sample Surveys & Random Rectangles



Notes: Sample Surveys & Random Rectangles

The Cluster Random Sample: To perform a cluster sample, the population is broken into smaller groups or clusters that are representative of the population. In other words, each cluster is a microcosm of the population. Once the clusters are identified, we number them and then randomly select one or more clusters. Once a cluster is selected, we then perform a census within the cluster, which is to say, that within each selected cluster we survey or sample every subject.

Cluster sampling tends to be difficult to perform with humans. For example, how would you divide Reagan up if you wanted to perform a cluster sample? Consider how you might divide the state of Texas or San Antonio into clusters. Keep in mind that each cluster needs to mirror the population, which means that each cluster is very similar to the other clusters.

You might be thinking that cluster sampling isn't useful or important. However, there are several instances in which cluster sampling is important. Here are a three scenarios:

Drawing Blood: When a lab needs to test a person's blood for a disease. They draw a small sample and then place it under the microscope. That small sample of blood under the microscope is a cluster and is representative of all of the blood in someone's body.

Baking: When you make cookies or cakes or brownies, you mix all of the ingredients together thoroughly and then you take a spoonful and taste it and make adjustments as necessary. That spoonful is a cluster and is representative of all the batter that is being made. I certainly could use a chocolate chip cookie about now. Hint. Hint…

Forrest Gump: "Life is like a box of chocolates. You never know what you're going to get." However, my expectation is that every box of Russell Stover's Chocolate Truffles is the same. So if I wanted to perform a cluster sample, I would randomly select a box, (this would be my cluster). Now remember to perform a cluster sample, I must perform a census within the chosen group. To perform a census in this scenario would mean that I would need to take a bite out of every truffle. Oh how wonderful. Incidentally, I may need to increase my sample size which would meant that I would need more boxes of chocolate and would need to eat all of the chocolate. It's a tough job, but someone's got to do it.

The rectangles have been broken into2 20 clusters of 5 which are representative of the population of rectangles. Number each cluster 1 to 20 and randomly select one of the clusters using the random number generator on your calculator

Calculator Commands:

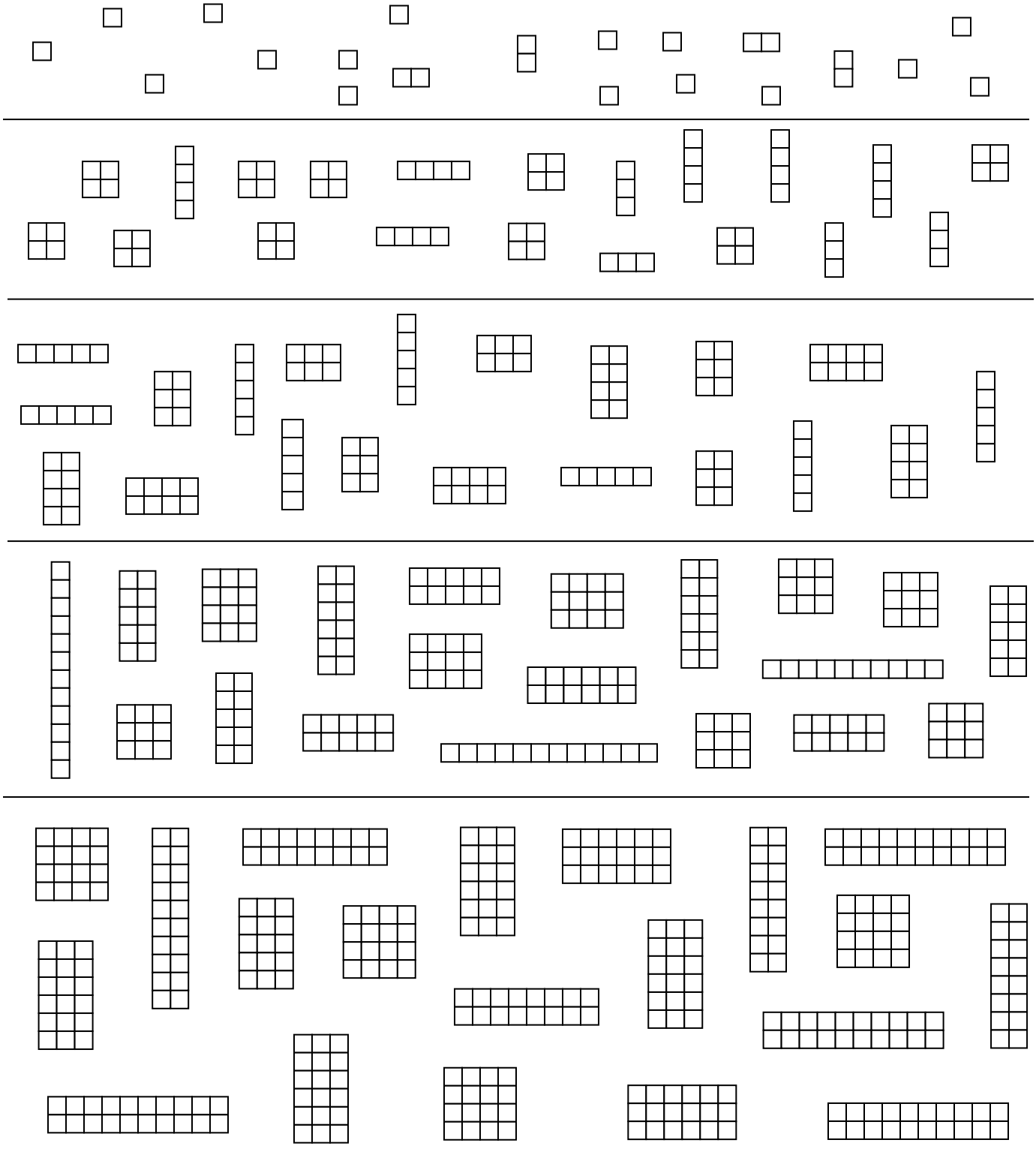
Press **MATH** → Highlight **PROB**
↓Highlight **8: randIntNoRep(**
Press **ENTER**
Lower: 1 Upper: 20 n: 1 Press **ENTER 2x's**

Locate the corresponding cluster and record the number of rectangles in each figure and find the mean of your sample

_____ _____ _____ _____ _____ mean = _____

Notes: Sample Surveys & Random Rectangles

Label each of the 5 sections of rectangles below based on the number of rectangles in each figure.



Number the figures in each section 1 to 20.

Notes: Sample Surveys & Random Rectangles

The Stratified Random Sample: To perform a stratified sample the population is divided into groups called strata based on a common variable or characteristic. We then select a random sample from within each group. Although it is not required, typically, the size of the random sample within each group is proportional to the variables representation within the population. It is important to note that the strata are not determined randomly. Randomness is introduced through the selection within the strata. Please understand that although we take a simple random sample within each strata, a stratified random sample **IS NOT** a special case of the simple random sample.

We take stratified samples for two important reasons. First, a stratified random sample allows us to ensure that each of the subgroups of interest within a population are represented in the sample. Secondly, we are able to isolate the impact of the common characteristic that determined the groups. Please note that unlike cluster samples (where each cluster is a microcosm of the population), the individual strata or groups are not representative of the population.

Dividing groups by gender is one of the most common ways we stratify and we do this because we believe that in general there are differences in males and females. However, my go to example for stratified sample is our high school. For Instance, let's say that we were trying to choose a theme song for homecoming and instead of performing a census and surveying everyone, we decide to survey the opinions of 100 students. We could do a simple random sample, however we run a risk of having chosen all freshmen in the sample. I don't think the seniors would be very pleased to have freshmen making decisions for them.

However, if we divided Reagan by grade level (freshmen, sophomore, junior and senior) and then performed a simple random sample within each grade level. We would be assured of having every grade level represented. In addition, since we know that 28% of the school is comprised of freshmen, 26% are sophomores, 23% are juniors and 23% are seniors, we could proportionally represent each grade level and our sample would be representative of the Reagan population.

Use the random number generator on your calculator to select 5 numbers for our sample.

Calculator Commands:

Press **MATH** → Highlight **PROB**
↓Highlight **8: randIntNoRep(**
Press **ENTER**
Lower: 1 Upper: 20 n: 1 Press **ENTER 2x's**

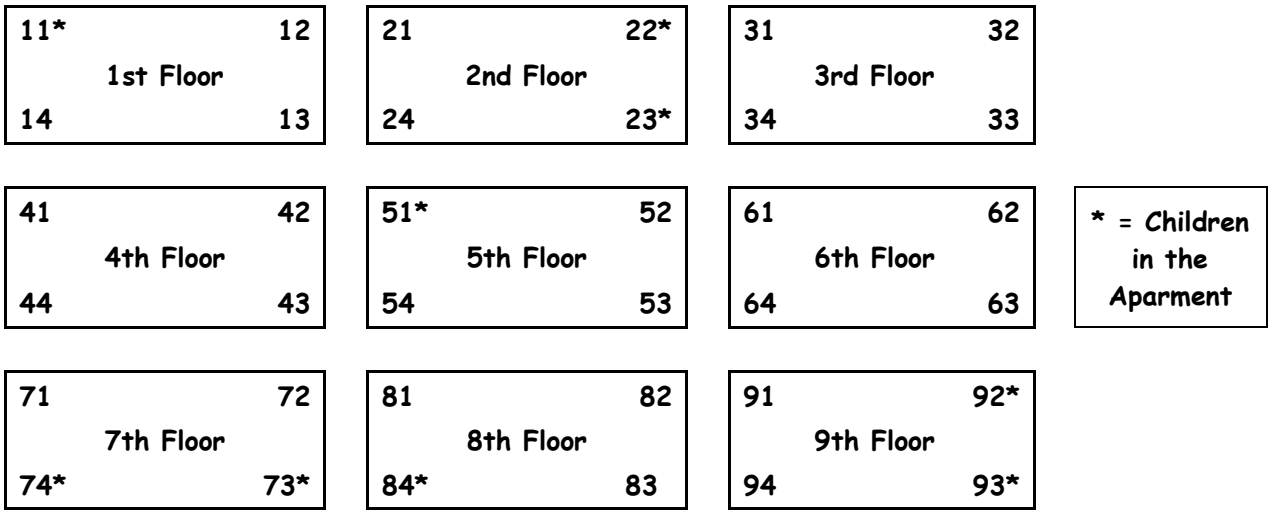
Locate the corresponding figure and record its size.
Note: Repeat the process 5 times (once for each strata)
Calculate the mean.

_____ _____ _____ _____ _____ mean = _____

Notes: Survey Scenarios

2011 Question 3: An apartment building has nine floors and each floor has four apartments. The building owner wants to install new carpeting in eight apartments to see how well it wears before she decides whether to replace the carpet in the entire building.

The figure below shows the floors of apartments in the building with their apartment numbers. Only the nine apartments indicated with an asterisk (*) have children in the apartment.



(a) For convenience, the apartment building owner wants to use a cluster sampling method, in which the floors are clusters, to select the eight apartments. Describe a process for randomly selecting eight different apartments using this method.

(b) An alternative sampling method would be to select a stratified random sample of eight apartments, where the strata are apartments with children and apartments with no children. A stratified random sample of size eight might include two randomly selected apartments with children and six randomly selected apartments with no children. In the context of this situation, give one statistical advantage of selecting such a stratified sample as opposed to a cluster sample of eight apartments using the floors as clusters.

(c) Why did we choose to stratify on apartments with and without children?

Notes: Survey Scenarios

2010 Form B Question 2: In response to nutrition concerns raised last year about food served in school cafeterias, the Smallville School District entered into a one-year contract with the Healthy Alternative Meals (HAM) company. Under this contract, the company plans and prepares meals for 2,500 elementary, middle, and high school students, with a focus on good nutrition. The school administration would like to survey the students in the district to estimate the proportion of students who are satisfied with the food under this contract.

Two sampling plans for selecting the students to be surveyed are under consideration by the administration. One plan is to take a simple random sample of students in the district and then survey those students. The other plan is to take a stratified random sample of students in the district and then survey those students.

(a) Describe a simple random sampling procedure that the administrators could use to select 200 students from the 2,500 students in the district.

(b) If a stratified random sampling procedure is used, give one example of an effective variable on which to stratify in this survey. Explain your reasoning.

(c) Describe one statistical advantage of using a stratified random sample over a simple random sample in the context of this study.

Notes: Survey Scenarios

2013 Question 3: An administrator at a large university wants to conduct a survey to estimate the proportion of students who are satisfied with the appearance of the university buildings and grounds. The administrator is considering three methods of obtaining a sample of 500 students from the 70,000 students at the university.

(a) Because of financial constraints, the first method the administrator is considering consists of taking a convenience sample to keep the expenses low. A very large number of students will attend the first football game of the season, and the first 500 students who enter the football stadium could be used as a sample. Why might such a sampling method be biased in producing an estimate of the proportion of students who are satisfied with the appearance of the buildings and grounds?

(b) Because of the large number of students at the university, the second method the administrator is considering consists of using a computer with a random number generator to select a simple random sample of 500 students from a list of 70,000 student names. Describe how to implement such a method.

(c) Because stratification can often provide a more precise estimate than a simple random sample, the third method the administrator is considering consists of selecting a stratified random sample of 500 students. The university has two campuses with male and female students at each campus. Under what circumstance(s) would stratification by campus provide a more precise estimate of the proportion of students who are satisfied with the appearance of the university buildings and grounds than stratification by gender?

Notes: Experimental Design

Well-designed and conducted observational studies are a great way to collect data and are the most common type of data collection. A well-designed observational study can be statistically significant and demonstrate a strong relation or correlation. However, observational studies are limited and are unable to prove causation. Only well-conducted experiments have the power to prove causation.

So what is an experiment?

An experiment: is a controlled study that requires a treatment be assigned to one or more groups. The effects of the treatments on the response variable are then compared. Experiments must satisfy the principles of **control, random assignment, and replication**.

Given the simplicity of the definition: you might be inclined to wonder why more experiments are not conducted. You probably will guess two reasons right away. Yup, good old time and money. I never seem to have enough of either, although I did get a 1% pay raise this summer. Another reason experiments are not performed more often is ethics. Ethics? Remember the definition states that treatments must be assigned or imposed. Let's assume that I want to measure the impact of smoking tobacco cigarettes on lung function. I don't think it is difficult understand that it would be unethical for me to assign one group of people to smoke three packs of cigarettes while assigning another group to smoke three cigarettes a day and a third group to was assigned to no tobacco products at all.

Assuming that we have time, money and no ethical issues, what are the critical components to conducting a well-designed experiment?

The Critical Elements necessary for a well-designed experiment are: **random assignment, replication and control.**

Random Assignment is the principle of randomly assigning subjects or experimental groups to various treatment groups. We use random assignment to equal out or mitigate the differences in subjects or experimental units that we are unable to control in our design. In other words, humans have too many different traits to "control for" or account for in a study. To make up for this lack of control, we use random assignment to equalize those differences across the experimental groups which avoids any biased judgement in the placing of subjects into a group. **There must be at least 2 treatment groups.** One of the groups may be a control group. (discussed on the next page).

Please note that random assignment is not random selection. In random selection, we randomly selected subjects from the population of interest and observed the responses. In random assignment, we may actually have volunteers for the study, but we use randomization to assign the treatments to the volunteers.

Replication in science refers to the ability of duplicating the results. In statistics we refer to replication as the imposition of the treatment on multiple subjects in order to gain confidence that the result can be generalized across the population and is not a one person anomaly.

Notes: Experimental Design

Control is the accounting for the impact of outside variables that are not of interest to us in our study. We design our experiments to remove or isolate the effects of external influences that we know will affect our study but are not of interest. We do this so that we can accurately measure the impact of the variables of interest. The following is a list of ways in which we exert control in our experiments: **Blocking, Blinding, and Control Groups.**

Blocking: Blocking is the separation of experimental units into groups with similar traits in order to control for the effect of outside variables that **we know** will have an impact or **confound** the results of the study. Confounding means that we are unable to determine which variable is causing the changes that we are observing. Blocking is not required, however if you block, always **block on the variable having the greatest impact** on the study.
Note: Blocking is to experiments as stratification is to surveys-**DO NOT** interchange the terms.

Blinding: In order to avoid a potential source(s) of bias, the subject or data collector or both are unaware as to the treatment that the subject is receiving. If both the subject and data collector are unaware of the treatment being assigned, the study is considered to be **double blind.**

Control Group: Typically a control group does not receive a treatment. The purpose of the control group is to provide a **baseline for comparison**. This allows us to determine whether the changes that are being observed can be attributed to uncontrolled for factors outside the parameters of our study. A **placebo** is often used as a method to establish a control group so that the subject and data collector believe that a treatment has been given.

The following scenario will be utilized to address the various methods utilized to control for the impact of outside variables that may impact the response, but are not the primary interest of the study.

Scenario: Researchers desire to test a new medication and treatment protocol for Alzheimer's disease. The new medication and treatment is thought to reverse the impact the disease has on memory function. Currently, there are no known side effects, but the drug has been only used on laboratory mice. As of this writing, no drug or treatment has been effective in reversing the effects of the disease.

Blocking: Because gender often impacts responses to medication, the researchers have chosen to block by gender.

Single Blinding: All of the patients in the study appear to be in the early stages of the Alzheimer's disease and are fully aware of what is going on. Consequently, the subjects are being told which treatment they are receiving as it would be depressing to learn that you were receiving a placebo or the old ineffective medication. The medication will be in containers marked A, B, and C and the patient will not be able to determine which medication they are receiving.

Double Blinding: The data collector will not be aware which medication the patient is receiving enabling the data collector to administer of the memory tests without bias.

Control Group: A Placebo is being administered to one of the patient groups. This will permit the researchers to be able to measure the progression of the disease and will let them know whether the changes being observed are due to the medication or are the result of some outside influence.

Notes: Observational vs. Experiment

Design an Observational Study and an experiment

Example: A study is to be designed to determine whether daily calcium supplements benefit women by increasing bone mass.

(a) How can an observational study be performed?

(b) How can an experiment be performed?

Which is More Appropriate a Study or Experiment?

Example: A study is to be designed to determine whether daily calcium supplements benefit women by increasing bone mass. Which is more appropriate an observational study or experiment and why?

Example: A study is to be designed to examine the life expectancies of tall people versus those of short people. Which is more appropriate an observational study or experiment? Why?

Example: A study is to be designed to examine GPA's of students who use marijuana regularly and those who don't. Which is more appropriate an observational study or experiment? Why?

Notes: Experimental Design Scenarios

1999 Number 3: The dentists in a dental clinic would like to determine if there is a difference between the number of new cavities in people who eat an apple a day and in people who eat less than one apple a week. They are going to conduct a study with 50 people in each group.

Fifty clinic patients who report that they routinely eat an apple a day and 50 clinic patients who report that they eat less than one apple a week will be identified. The dentists will examine the patients and their records to determine the number of new cavities the patients have had over the past two years. They will then compare the number of new cavities in the two groups.

(a) Why is this an observational study and not an experiment?
(b) Explain the concept of confounding in the context of this study. Include an example of a possible confounding variable.
(c) If the mean number of new cavities for those who ate an apple a day was statistically significantly smaller than the mean number of new cavities for those who ate less than one apple a week, could one conclude that the lower number of new cavities can be attributed to eating an apple a day? Explain.

Notes: Experimental Design Scenarios

2006 B Number 5 When a tractor pulls a plow through an agricultural field, the energy needed to pull that plow is called the draft. The draft is affected by environmental conditions such as soil type, terrain, and moisture.

A study was conducted to determine whether a newly developed hitch would be able to reduce draft compared to the standard hitch. (A hitch is used to connect the plow to the tractor.) Two large plots of land were used in this study. It was randomly determined which plot was to be plowed using the standard hitch. As the tractor plowed that plot, a measurement device on the tractor automatically recorded the draft at 25 randomly selected points in the plot.

After the plot was plowed, the hitch was changed from the standard one to the new one, a process that takes a substantial amount of time. Then the second plot was plowed using the new hitch. Twenty-five measurements of draft were also recorded at randomly selected points in this plot.

(a) What was the response variable in this study? Identify the treatments. What were the experimental units?

(b) Given that the goal of the study is to determine whether a newly developed hitch reduces draft compare to the standard hitch, was randomization used properly in this study? Justify your answer.

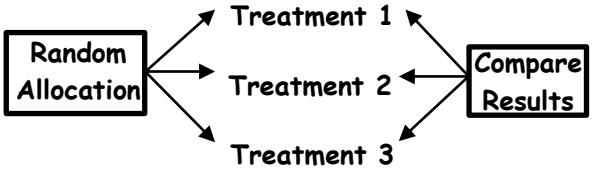
(c) Given that the goal of the study is to determine whether a newly developed hitch reduces draft compare to the standard hitch, was replication used properly in this study? Justify your answer.

(d) Plot of land is a confounding variable in this experiment. Explain why.

Recipe for Success: Completely Randomized Design Q 1.2

Completely Randomized Design: Subjects are assigned treatments by chance alone.

1. Draw the Diagram



- Draw the RA Box - Random Assignment
- Add Treatments – Label Accurately
- Draw the Compare Results Box

2. Explain the Random Allocation (Random Assignment)

- Place names in a hat, mix and draw
- Flip a Coin
- Assign a number and place in a hat, mix and draw
- Assign a number and use a random number generator (be certain to address repeats)

3. What is Being Assigned?

- Examine the Treatments
- Name Each Treatment Group (could be more than 1)

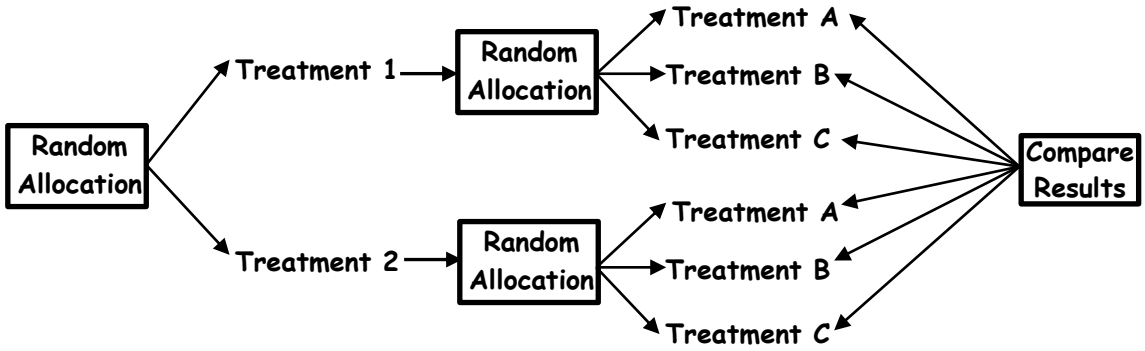
This is the Explanatory Variable(s) or Factor

4. Identify the Results to Compare

- Specifically state what is being compared

This is the Response Variable

Multi-Variable Completely Randomized Design



Explanatory Variables, Levels & Treatments
(Based on the Diagram)

- **1 Numerical Variable: 2 Levels 1 & 2**
- **1 Letter Variable: 3 Levels A, B & C**
- **6 Treatments: 2 x 3 = 6 Treatments**
  - *Treatment 1 A*
  - *Treatment 1 B*
  - *Treatment 1 C*
  - *Treatment 2 A*
  - *Treatment 2 B*
  - *Treatment 2 C*

Notes: Experimental Design Scenarios

2007 Question 2: As dogs age, diminished joint and hip health may lead to joint pain and thus reduce a dog's activity level. Such a reduction in activity can lead to other health concerns such as weight gain and lethargy due to lack of exercise. A study is to be conducted to see which of two dietary supplements, glucosamine or chondroitin, is more effective in promoting joint and hip health and reducing the onset of canine osteoarthritis. Researchers will randomly select a total of 300 dogs from ten different large veterinary practices around the country. All of the dogs are more than 6 years old, and their owners have given consent to participate in the study. Changes in joint and hip health will be evaluated after 6 months of treatment.

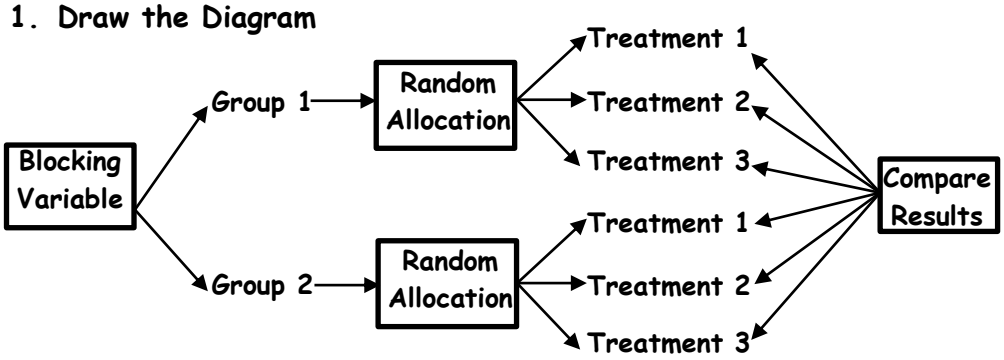
(a) What would be an advantage to adding a control group in the design of this study?

(b) Assuming a control group is added to the other two groups in the study, explain how you would assign the 300 dogs to these three groups for a completely randomized design.

(c) Rather than using a completely randomized design, one group of researchers proposes blocking on clinics, and another group of researchers proposes blocking on breed of dog. How would you decide which <u>one</u> of these two variables to use as a blocking variable?

Recipe for Success: Randomized Block Design(s) Q1.2

1. Draw the Diagram



1. Draw the Blocking Variable Box
2. Label Groups Accurately
3. Draw Random Assignment Boxes
4. Label Treatments
5. Draw the Compare Results Box

2. Explain The Block
- Define the variable that is being blocked
- List the Groups in the Block

3. Explain the Random Allocation (Random Assignment)
- Place names in a hat, mix and draw
- Flip a Coin
- Assign a number and place in a hat, mix and draw
- Assign a number and use a random number generator (be certain to address repeats)

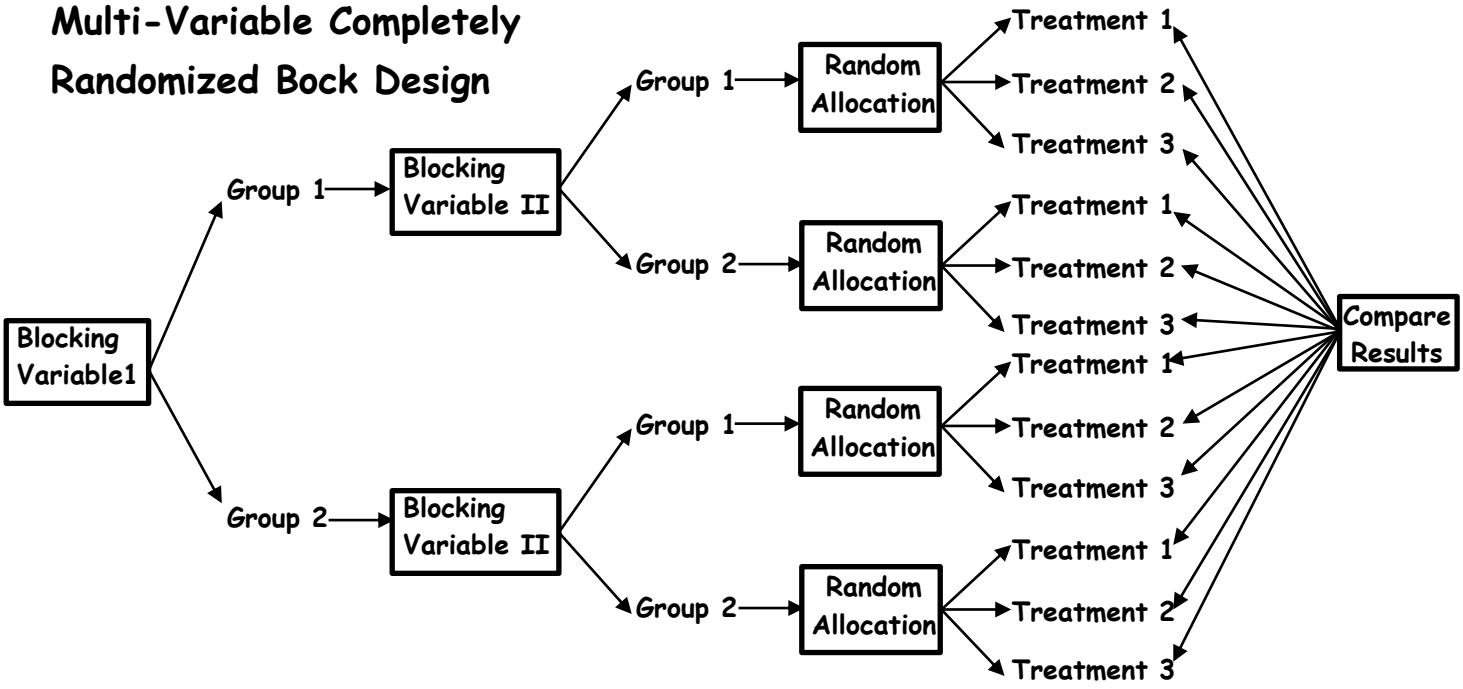
4. Identify the Results to Compare
- Specifically state what is being compared

This is the Response Variable

5. What is Being Assigned?
- Examine the Treatments
- Name Each Treatment Group (could be more than 1)

This is the Explanatory Variable(s) or Factor

Multi-Variable Completely Randomized Bock Design



Explanatory Variables, Levels Treatments
(Based on the Diagram)

- **Blocking Variable 1: 2 levels**
- **Blocking Variable II: 2 Levels**
- **Numerical Variable: 3 levels**
- **Treatments: 2 x 2 x 3 = 12 Treatments**

Notes: Experimental Design Scenarios

2000 Number 5 High cholesterol level in people can be reduced by exercise or by drug treatment. A pharmaceutical company has developed a new cholesterol-reducing drug. Researchers would like to compare its effects to the effects of the cholesterol-reducing drug that is currently available on the market. Volunteers who have a history of high cholesterol and who are currently not on medication will be recruited to participate in a study.

(a) Explain how you would carry out a completely randomized experiment for the study.
(b) Describe an experimental design that would improve the design in (a) by incorporating blocking.
(c) Can the experimental design in (b) be carried out in a double blind manner? Explain.

Recipe for Success: Matched Pair Design Q1.2

(Each Group/Individual receives 2 Treatments)

1. Draw the Diagram

1. Draw the Random Assignment Box
2. Add Treatments (I and II) Label Accurately
3. Add Treatments (II and I) Label Accurately
4. Draw the Compare Results Box

2. Explain the Reason for utilizing a Matched Pairs Design

- To control for the variation within an individual or subject
- Give a real life reason to control for variation by individual

I. Lifestyle differences
(Water Proof Boots)

II. Significant Variation within each person

(Susceptibility to Mosquito Bites)
(Reaction to Medication)
(Reaction time based on dominant and non-dominant hand)

III. Control for unknown variables
(Each plot of land receives 2 plants)

3. Explain the Random Allocation (Random Assignment)

- Place names in a hat, mix and draw
- Flip a Coin
- Assign a number and place in a hat, mix and draw
- Assign a number and use a random number generator (be certain to address repeats)

4. Identify the Results to Compare

- Specifically state what is being compared

This is the Response Variable

5. What is Being Assigned?

- Examine the Treatments
- Name Each Treatment Group
- Give the order of the Treatments within each group

This is the Explanatory Variable(s) or Factor

Notes: Experimental Design Scenarios

2005 B Number 3 In search of a mosquito repellent that is safer than the ones that are currently on the market, scientists have developed a new compound that is rated as less toxic than the current compound, thus making a repellent that contains this new compound safer for human use. Scientists also believe that a repellent containing the new compound will be more effective than the ones that contain the current compound. To test the effectiveness of the new compound versus that of the current compound, scientists have randomly selected 100 people from a state.

Up to 100 bins, with an equal number of mosquitoes in each bin, are available for use in the study. After a compound is applied to a participant's forearm, the participant will insert his or her forearm into a bin for 1 minute, and the number of mosquito bites on the arm at the end of that time will be determined.

(a) Suppose this study is to be conducted using a completely randomized design. Describe a randomization process **and identify an inference procedure for the study.**

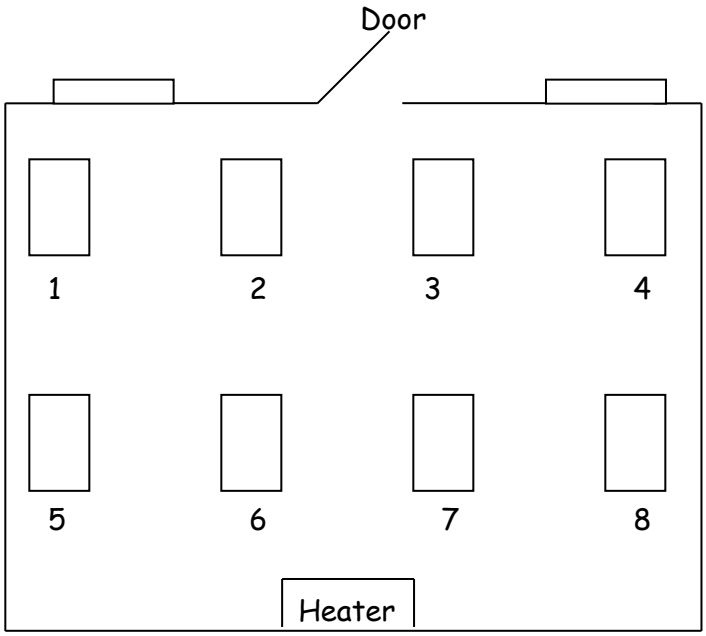
(b) Suppose this study is to be conducted using a matched-pairs design. Describe a randomization process **and identify an inference procedure for the study.**

(c) Which of the designs, the one in part (a) or the one in part (b), is better for testing the effectiveness of the new compound versus that of the current compound? Justify your answer.

Notes: Experimental Design Scenarios

1997 Question 2: A new type of fish food has become available for salmon raised on fish farms. Your task is to design an experiment to compare the weight gain of salmon raised over a six-month period on the new and old types of food. The salmon you will use for this experiment have already been randomly placed in eight large tanks in a room that has a considerable temperature gradient. Specifically, tanks on the north side of the room tend to be much colder than those on the south side. The arrangement of tanks is shown on the diagram below.

Describe a design for this experiment on weight gain that takes into account this temperature gradient.



Notes: Qualitative Data

Now that we have collected the data, how do we make sense of it? First, we need to determine the type of data that was collected. Is the data qualitative or is it quantitative?

Qualitative Data/ Categorical Data: Data that is used to describe an attribute (color, smell, how much pain you feel). **Can categorical data be numerical?**

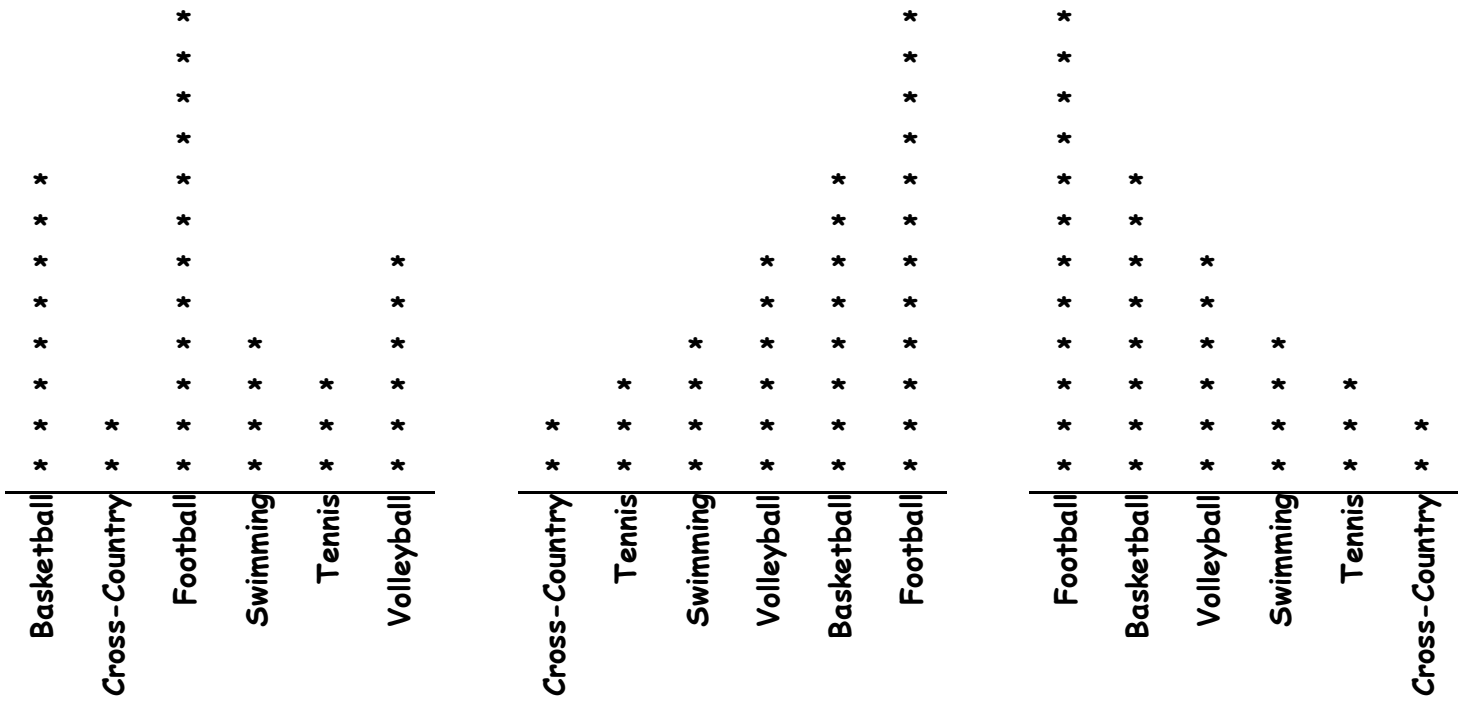
Quantitative Data: Data that can be measured and expressed numerically (example: height, weight, g.p.a, age. etc).

We will begin the section on displaying and describing data talking about qualitative data only because there it is less involved. Because qualitative data is not numerical in nature, we are unable to talk about the mean (average), median (the middle) or the range (largest-smallest). We may talk about the mode or modes which is the category(s) that occurs most often.

You have heard it said, that a picture is worth a thousand words and that is definitely the case with data. In the case of qualitative data the pictures or graphs that are primarily used are: dot-plots, bar-charts, pie-charts and segmented bar-charts.

Scenario 1: 35 students were asked to choose their favorite fall sport. The selections were as follows: swimming (4), basketball (8), football (12), tennis (3), volleyball (6) and cross-country (2).

Three dotplots have been created from the collected data. Which is correct and why?



Does the order matter for qualitative data? Why or why not?

Note: For categorical/qualitative data **DO NOT** describe shape, center, spread, or unusual features. **DO** talk about the mode-the group or category with the most.
Create three bar-charts for the scenario above. **Note:** the columns should not touch on bar charts.

Notes: Types of Categorical Displays

Regardless as to whether or not a graph represents categorical or quantitative data, it is important to not only accurately display the data, but to provide a **title**, **label the axes** and **provide a key** or legend. Failure to label has resulted in zero points being awarded for an otherwise perfectly drawn display. In addition to the dotplot displayed on the prior page, the following is a list of some of the more common categorical displays.

Bar Chart: A graph that presents categorical data with rectangular bars with heights or lengths proportional to the values that they represent. Because Bar charts represent categorical data, order does not matter and the bars should **NOT** touch one another. Bar lengths may convey actual data counts or they may represent relative frequencies/percents. If the bars represent actual counts or frequencies, then we know the exact number represented by the bar in the chart. However, if the bar chart is displaying relative frequencies (Percents), then we can look at the area of each bar and know its proportion of the sample relative to the other bars, but we will not have insight as to the actual number contained within the bar of the sample.

Bar Chart vs. Histogram: A histogram looks like a bar chart but is used to display quantitative data where order does matter and the bars must touch one another. We will spend more time discussing histograms in our discussion about quantitative data.

Pie Charts: A circle graph which is sub-divided into sectors based on their proportion to the whole. Please be aware that these are relative frequencies (proportions) and do not provide insight as to the number/size of the sample.

Segmented bar chart: A stacked bar chart in which each column is divided into segments which are proportional in size to that segment's representation within the population. Please be aware that these are relative frequencies (proportions) and do not provide insight as to the number/size of the sample.

Independence: Two events are considered to be independent, if the probability of one does not impact the probability of the other. If categories are independent then they are proportional. **Note:** We will continue develop the concept of independence throughout the remainder of this course.

Scenario 2: The following table provides the results which relates a person's IQ to a person's dominant hand. The table is illustrative of the proportional nature of independent categories.

Actual Counts

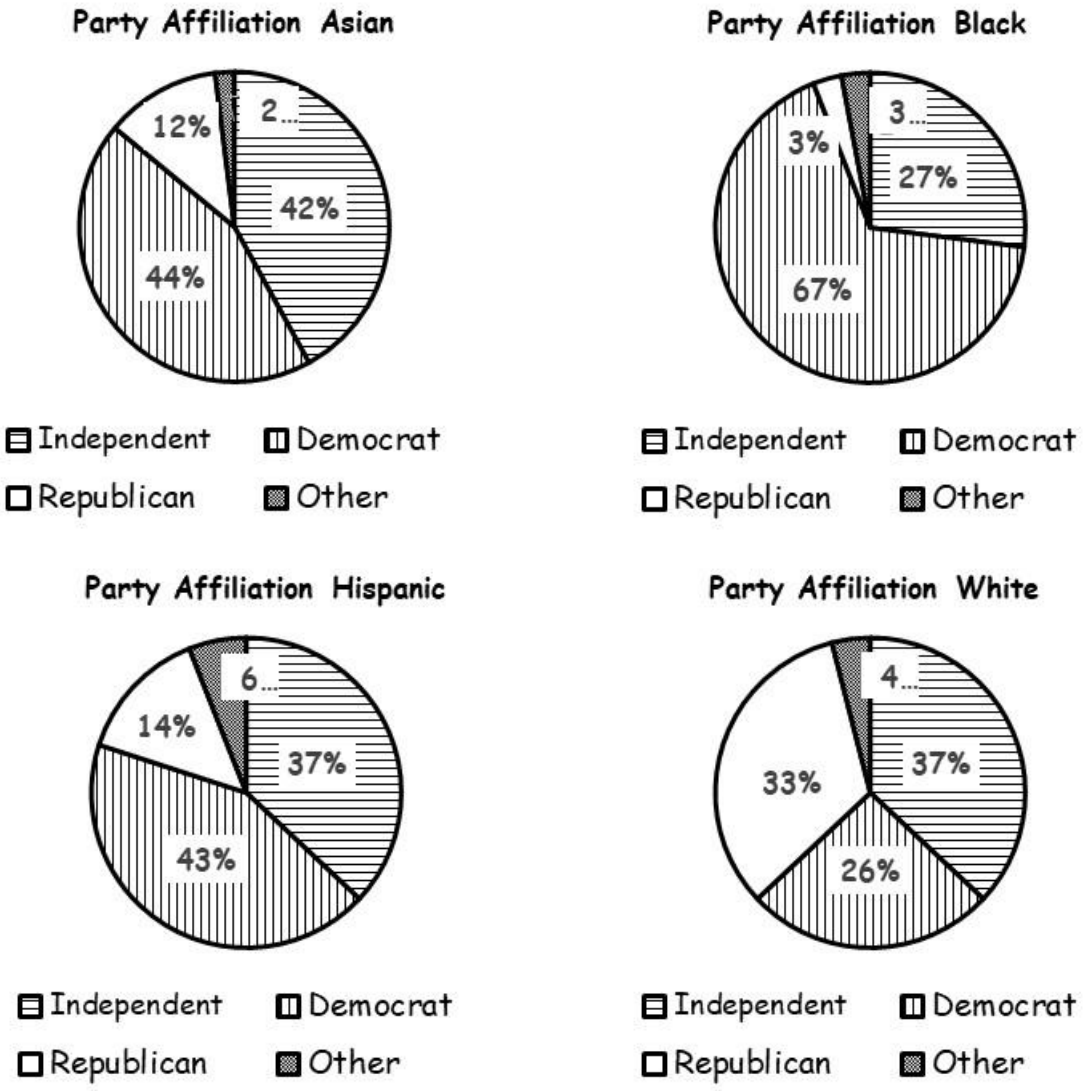
	Right Handed	Left Handed	Total
High IQ	190	10	
Normal IQ	1,710	90	
Total			

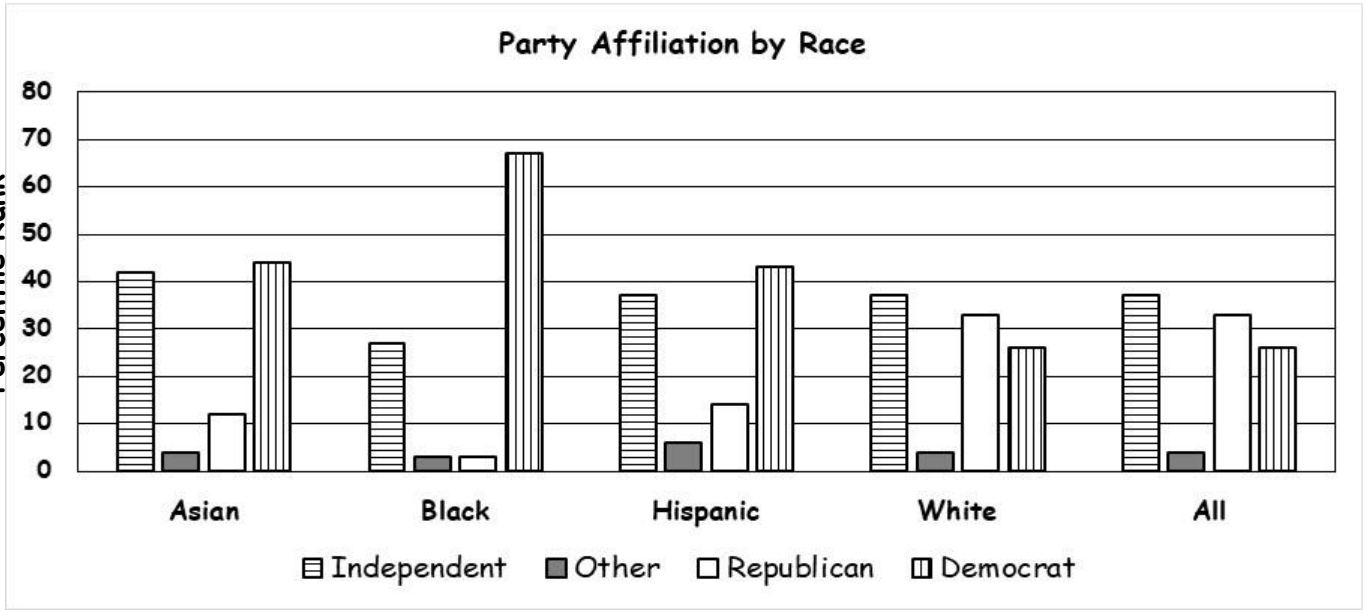
Proportionality

	Right Handed	Left Handed	Total
High IQ			
Normal IQ			
Total			

Notes: Pie Charts vs. Bar Charts

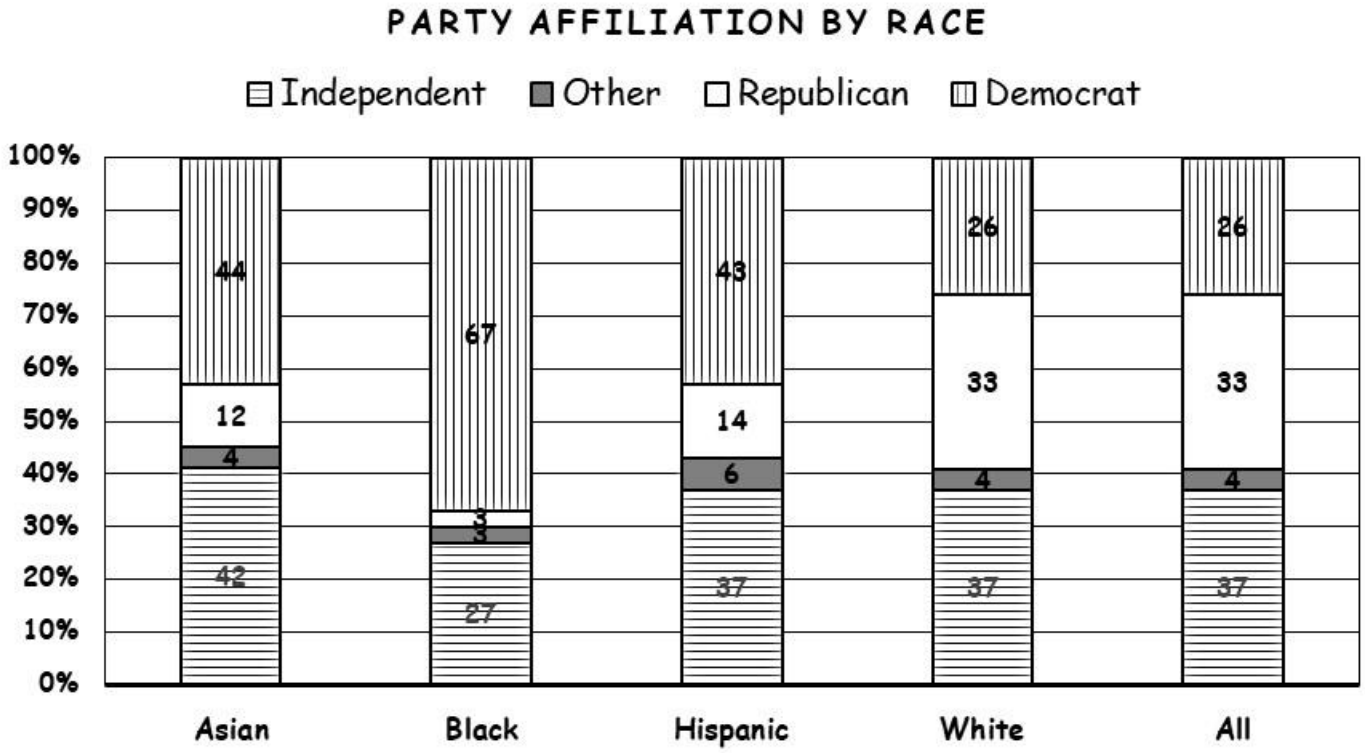
Scenario 3: The bar & pie charts represent a 2017 Pew Research poll of political affiliation by race.





Notes: Segmented Bar Charts

Scenario 3 Continued: The following segmented bar chart represents the same 2017 Pew Research poll of political party affiliation by ethnicity/race as the pie chart.



Questions: Using the segmented bar chart above to answer the following:

1. Can we say that there are more than twice as many Black Democrats as there are Black Independents? Justify your response.

2. Can we say that there are twice as many Black Democrats as there are White Republicans? Justify your response.

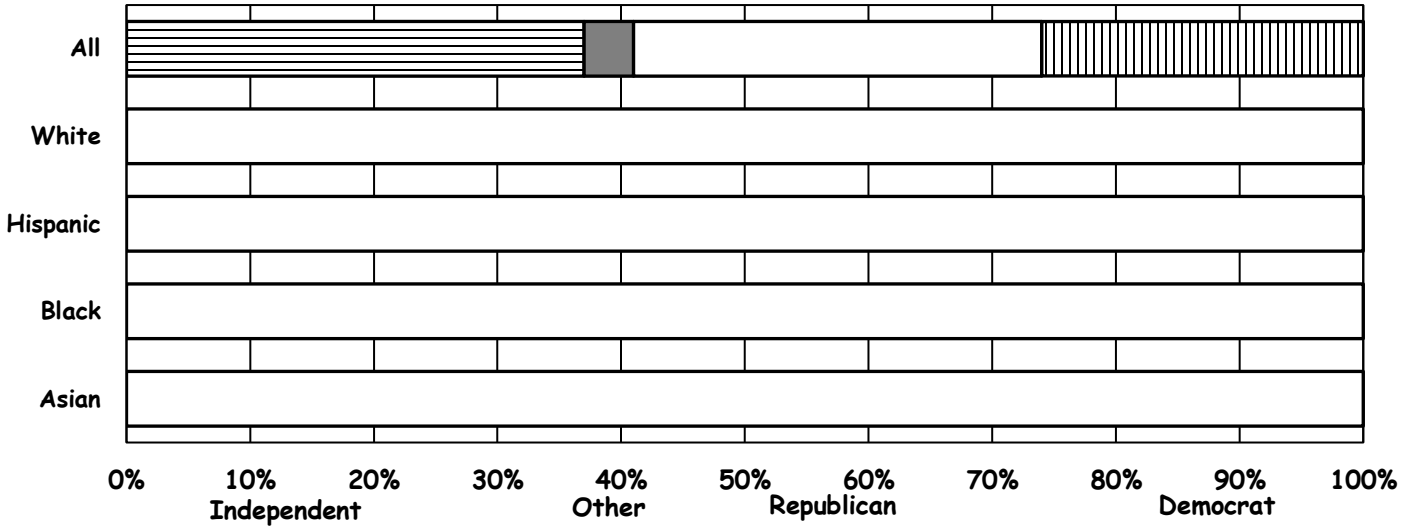
3. Can we say that the number of Hispanic independents is about the same as the number of White Independents? Justify your response.

4. "Does political affiliation appear to be related to race?" Another way to ask the question is: "Are political party affiliation and race independent of each other?" More simply put, if the events are independent then the distribution of political affiliation would be the same for each race.

Notes: Independence

Scenario 3 Continued: Based on the 2017 Pew Research data, party affiliation and race are not independent. However, for this exercise, assume that they are and the distribution of party affiliations follows that of the "All Race" distribution. Create a key and fill in the segments for each race.

Party Affiliation By Race



Scenario 5: Find the table value that results in perfect independence.

133	95
105	

Scenario 4: Given that grades in statistics are independent of gender, complete the table below.

	Female	**Male**	
A's	**30**		
B's	**50**		
C's	**20**		

Notes: Two-Way Contingency Tables

Two Way Contingency Tables are useful in showing the relationships between 2 Categorical Variables and provide the frequency of occurrence (distribution) across each variable. The is only a brief introduction to 2-way tables. A thorough investigation of 2-way tables will be conducted during the probability section of this course.

Marginal Frequencies: Are the number of times a single variable resulted in a particular outcome. A Marginal frequency is the total of as single row or single column.

Joint Frequencies: Represent the intersection of a row and a column and provide the number of times that two separate variables yielded a particular outcome (this and that) both occurred. The key work for a joint frequency is **AND**.

Scenario 6: The table below provides the frequencies and relationship between Cholesterol level & Heart Attack Severity. Complete the table.

Heart Attack Severity vs. Cholesterol Level

	Low Cholesterol	Medium Cholesterol	High Cholesterol	Total
Non-Fatal Heart Attack	29	17	18	
Fatal Heart Attack	19	20	9	
Total				

Questions: Complete the table and answer the following:

How many variables are there? Name them

How many levels are there for each variable? Name them with their associated variable.

What is the sample size represented in the table?

List the joint frequencies.

List the marginal frequencies.

Are the variables independent? Justify your response?

Notes: Two-Way Contingency Tables

Scenario 6 Continued: Please use the table on page 42 to answer the following:

What percent of people in the sample had a fatal heart Attack? Is this a marginal or joint probability?

What percent of people in the sample have low Cholesterol? Is this a marginal or joint probability?

What percent of people in the study had a high cholesterol level and had a non-fatal heart attack? Is this a marginal or joint probability?

What percent of people had a medium cholesterol level and had a fatal heart attack? Is this a marginal or joint probability?

Conditional probability means that we have added a condition or restriction. As a consequence of the condition we are looking at a portion of the sample and are no longer looking at the entire sample. Given is the key word used to identify a conditional restriction and that word lets us know which marginal frequency we are being limited to. Because we are no longer using the total sample the denominator becomes the marginal frequency and not the sample total.

$$\frac{\text{Joint Frequency}}{\text{Marginal Frequency}} = \frac{\text{"and" "both"}}{\textbf{Given}} = \frac{\text{intersection}}{\textbf{Given}}$$

Given that a person had a fatal heart attack, what is the probability that they had high cholesterol?

Given that a person had a low cholesterol, what is the probability that they had a fatal heart attack?

Given that a person had a moderate cholesterol level, what is the probability that they had a non-fatal heart attack?

Please complete the table using percentages instead of frequencies

Heart Attack Severity vs. Cholesterol Level

	Low Cholesterol	Medium Cholesterol	High Cholesterol	Total
Non-Fatal Heart Attack				
Fatal Heart Attack				
Total				

Notes: Simpson's Paradox

Simpson's Paradox: when the results from combined grouping appears to contradict the results from the individual groupings. Simpson's Paradox arises when two or more sub-groups are combined to form a single group and there exists significant differences in the sizes of the sub-groups and the proportions in each group differ.

Test results for students from two teachers at a low performing school are summarized in the table below. Based on the table below make a determination as to which teacher is better at helping their students succeed. Justify your response.

Passed	130	125	
Failed	15	20	
Total			

Further test score information became available in which the students were broken into two groups, the group that failed the prior year's test and those that had passed the prior year's test.

Passed the Prior Year's TAKS Test

	Teacher A	Teacher B	Total
Passed	110	30	
Failed	5	0	
Total			

Failed the Prior Year's TAKS Test

	Teacher A	Teacher B	Total
Passed	20	95	
Failed	10	20	
Total			

Based on the information contained in the subsequent tables which teacher did a better job? Justify your response and reconcile any differences with your prior response.

What is the lurking variable in this situation?

Notes: C.U.S.S. & B.S.

We have been discussing non-numerical data or qualitative data which is used to describe attributes: gender, color, ethnicity, physical condition and many other things. We also recognized that while qualitative data may be non-numerical in that order doesn't really matter, we sometimes assign numbers for coding purposes, such as the UPC product code, or a social security number or girls are considered 1's and guys are considered to be zeroes. However, terms like mean or median and range made no sense, because there was no order. For instance, we could take the average of all the hair color in the room and come up with deep purple. We wouldn't average the girls and guys and come up with a "shim."

Now we are moving on to discuss Quantitative data which is truly numerical.

Quantitative Data: Data that can be measured and expressed numerically (example: height, weight, number of pets, g.p.a, age. etc).

In this case order does matter: Consider your class rank, or how much you can lift, or how fast you are, or how much money you make (I should say earn). In these cases, order absolutely matters. Of course, because you need some more definitions to contend with Quantitative data can be divided into two separate groups discrete and continuous. For now, we are going to only introduce the definitions, but we will explore both in detail.

Discrete-quantitative data that can be counted (example: the number of students)

Continuous- quantitative data that could take on any fractional value (volume, height, weight)

Prior to developing a common language to describe the distributions there are some very simple computations that can be made to help summarize the data. Let's begin by inputting the following data into the calculator:

9, 8, 7, 6, 5, 4, 3, 2, 1, -10

Calculator:

Press **STAT**; highlight **EDIT**; & press **ENTER** *(enter the data into L_1)*
Press **2nd Mode/Quit**
Press **STAT** → highlight ***CALC*** highlight **1:1-Var Stats** & press **ENTER**
List: press **2nd** & press **1**; press **ENTER**
Frequency: Press **CLEAR**; press **ENTER**; press **ENTER**

- $\bar{\mathbf{x}}$ = the mean (technically $\bar{\mathbf{x}}$ is the symbol for the sample mean) and is the sum of the values divided by the sample size
- $\boldsymbol{\Sigma \mathbf{x}}$ = the sum of all of the values
- $\boldsymbol{\sigma_x}$ = the standard deviation of the population *(we will discuss the calculations at a later date)*
- $\boldsymbol{S_x}$ = the standard deviation of the sample *(we will discuss the calculations at a later date)*
- $\boldsymbol{n}$ = the sample size

5 Number Summary:

- minX = the minimum value of the data set-(it may or not be an outlier)
- $\boldsymbol{Q_1}$ denotes the 1st quartile and 25% of the data set fall below this value
- Med = the middle most value when the data is listed least to greatest. 50% of the data set fall below this value
- $\boldsymbol{Q_3}$ denotes the 3rd quartile and 75% of the data set fall below this value.
- maxX = the maximum value of the data set-(it may or not be an outlier)

Notes: C.U.S.S. & B.S.

As we work through this section on quantitative data, we will learn to create data displays and we will develop a common language to describe the distributions of the data. We will begin with developing the common language which follows the acronym **C.U.S.S.**

C: Measures of the Middle (mean and median)

U: Unusual Features (gaps, clusters and outliers)

S: Shape (skewed left, skewed right, mound, symmetric, uniform, bi-modal and multi-modal)

S: Spread (variation, range, interquartile range, variance and standard deviation)

When describing or comparing distributions you will need to address all four of the parts of **C.U.S.S.** and you will need to B.S. those in your descriptions, that is to say you must **B**e **S**pecific.

When it comes to being able to C.U.S.S. appropriately, all parts are important and should be addressed. However, I will give you a rank ordering of importance.

1. **Center and Spread:** These must always be addressed. Failure to do so will result in no points being awarded. (... and may God have mercy on your soul)— Billy Madison
2. **Shape:** If you want to receive full credit, you should be discussing the shape of the distribution
3. **Unusual Features:** If there exist unusual features they need to be addressed. If they are not addressed, you can expect point deductions.

C: Measures of the Middle

- **Mean:** The average of all of the values (the mean is impacted by outliers and non-symmetric distributions). The mean is the balance point on a scale or see-saw.
  Calculation: Sum all of the values and divide by **n** the number of values summed.
- **Median:** the middle-most number (median is not impacted by outliers and non-symmetric distributions). 50% of the values are larger and 50% are smaller than the median.
  Calculation: Arrange the numbers smallest to greatest and find the middle number.

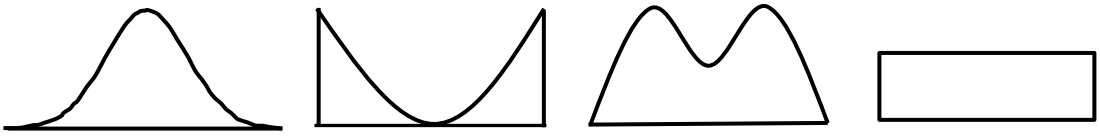
U: Unusual Features

- **Gap:** a section in the distribution without a data point
- **Clusters:** Clusters are distinct groups of data as in two mounds. (**Note:** if clusters exist the centers spread and shape of the clusters must be given)
- **Outliers-**data points that are either too large or too small. In this course, we identify outliers utilizing 2 methods:
  1. More than 2-3 standard deviations above or below the mean. (We will learn about standard deviations, a measure of variation, in the near future.)
  2. More than 1.5 Interquartile ranges below the 1st quartile or more than 1.5 interquartile ranges above the 3rd quartile. (We will discuss the interquartile range and the 1st and 3rd quartile when we discuss box plots)

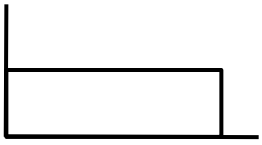
Notes: C.U.S.S. & B.S.

S: Shape

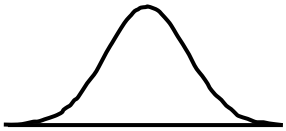
- **Symmetric:** The vertical line can divide the data into two matching mirror images. If the data is symmetric the mean and median will be equal.

  Examples: 

- **Uniform:** A graph that is approximately the same height.

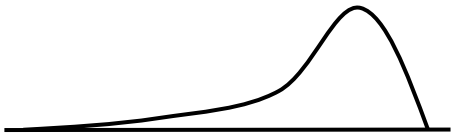
  Example: 

- **Mound Shape:** The graph is symmetric with most of the data in the center of the graph. The data density diminishes as you move towards each tail. The mean, median and mode are all equal.

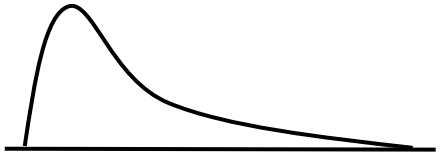
  Example: 

Because data is rarely if ever perfect, we need to qualify the above shapes as approximately symmetric or reasonably symmetric or reasonably mound shaped or approximately uniform.

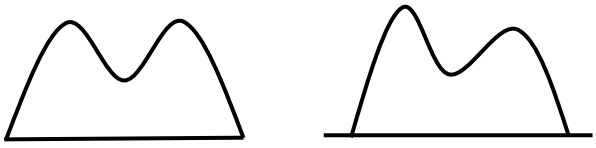
- **Skewed Left:** The tail is to the left and the mean is less than the median. The mean is to the left of the median. (imagine pulling on a sticky wad of gum towards the left)

  Example: 

- **Skewed Right:** The tail is to the right and the mean is greater than the median. The mean is to the right of the median. (imagine pulling on a sticky wad of gum towards the right)

  Example: 

- **Bi-modal:** The distribution has two distinct peaks (modes). The peaks may not be equal in height.

  Examples: 

Notes: C.U.S.S. & B.S.

S: Spread

- **Range:** The range provides an idea as to how spread out the data is by subtracting the smallest value from the largest value. The range is a singular value and is never negative. Because the range uses the largest and smallest value it is greatly impacted by outliers.

- **Interquartile Range:** Then interquartile range or IQR gives an idea as to how spread out the data is by focusing on the middle 50% of the data and is computed by subtracting the value of the 1st quartile from that of the 3rd quartile. We express it in this manner IQR= $Q_3 - Q_1$. Because the interquartile range focuses on the middle 50% of the data it is not impacted by outliers. IQR is very useful for skewed distributions.

- **Variance:** The variance takes an average of the data distances from the mean. Because some values are below the mean the distances are negative and obviously some of those distances are greater than the mean and are positive. As a consequence, the distances are squared and then summed. σ^2 is the symbol for the true population variance.
  Remember: we use means as a measure of center when the data tends to be mound shaped and symmetric. Because variance uses the mean in the calculation we tend to reserve the use of the variance for distributions that are symmetric and mound shaped.

  The formula for variance of the entire population is $\sigma^2 = \sum \frac{(x_i - \mu)^2}{n}$ where μ is the true population mean.

  Unfortunately we rarely know the true population mean and have to rely on the sample mean which has the symbol $\bar{x}$. The symbol for variance of the sample is s^2 and the formula is as follows: $s^2 = \sum \frac{(x_i - \bar{x})^2}{n-1}$ You may notice that the sample differences are divided by n-1 this is done because we are using the sample mean, $\bar{x}$ an estimate of the true population mean, in our calculation.

- **Standard Deviation:** The standard deviation is just the square root of the variance. Because of advances in technology we are able to easily take square roots and as consequence we tend to talk about standard deviations more than we do variances. The symbol for the standard deviation of the population is σ and the formula is: $\sigma = \sqrt{\sum \frac{(x_i - \mu)^2}{n}}$. As expected, the symbol for the standard deviation of the sample is just s and the formula is: $s = \sqrt{\sum \frac{(x_i - \bar{x})^2}{n-1}}$. We will typically say something is so many standard deviations from the mean. Because we are relying on the mean, we usually reserve standard deviation for data that is mound shaped. The smaller the standard deviation the closer the data is to the mean.

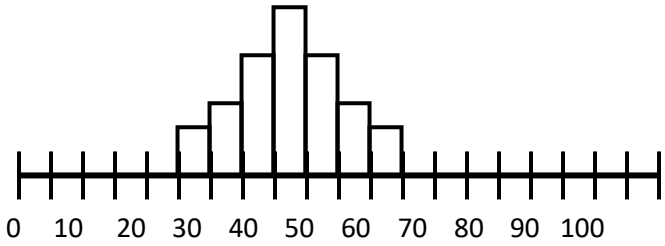
- Z-scores: A Z-score is a ratio that provides a measure as to how far a value is from the mean and takes into account both the center and the dispersion of the data. Z-scores act as a ruler and can be used to compare different shaped distributions the basic z score formula is $z = \frac{x - \mu}{\sigma}$ where **z is the number of standard deviations a value lies from the mean.**

Date:_____ Compiled by: Mr. Loren L. Spencer

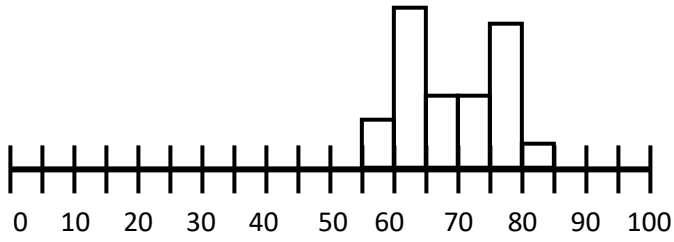
Practice: C.U.S.S. & B.S.

Directions: Describe the following graphs. Make certain that you C.U.S.S. We will worry about computations later.

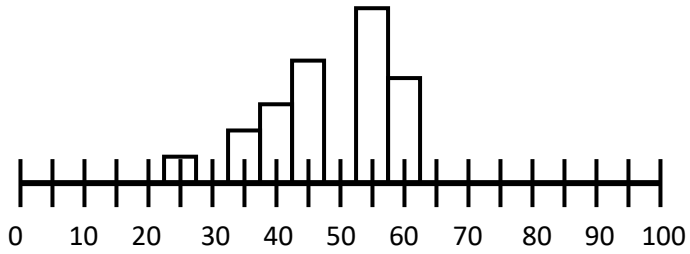
1.



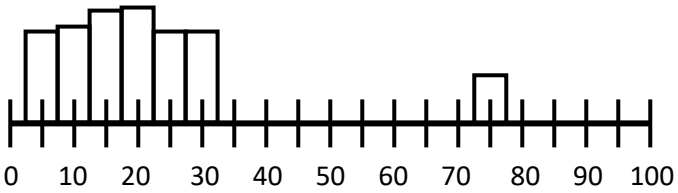
2.



3.



4.



Notes: Creating Stem and Leaf Plots

One of the most basic quantitative graphs is the stem and leaf plot which from this point forward I will refer to as a stemplot. Data values are placed in order from least to greatest and each data value is divided into both a stem and leaf component. The leaves are comprised of furthest digit to the right while the stem is comprised of the remaining digits. To visualize the data the leaves of data values are stacked onto a common stem. The advantage to this type of data display is that it maintains all of the original data values and it provides an idea of center shape, spread and any unusual features in a data set. Unfortunately, this type of graph becomes unwieldly with large data sets.

Example: The following table provides a list of weights in kilograms for the forwards and backs of a rugby team that toured parts of New Zealand.

Backs	120	85	90	83	90	81	110	84	113	77
	75	87	100	94	91	79	100	82	94	102

Forwards	128	100	89	105	105	101	115	108	114	99
	98	104	108	101	100	99	110	105	105	115

Create a Stemplot for the Backs

Step 1: Input the data into L_1 and follow the calculator commands below:

2^{nd} STAT
→ OPS
Highlight Sort A
Enter
2^{nd} "column number"
Enter
STAT EDIT ENTER

Step 2: Create a legend

Step 3: Draw a vertical Line

Step 4: Input the Stems

Step 5: Input the leaves

Input the data for the forwards into L_2. Repeat the above Steps to create back to back Stemplots. Compare and contrast the 2 groups.

Notes: Creating Histograms

A histogram is another type of frequency distribution whose class/bar widths have a height that is proportional to the frequency of the values in that class. Histograms are useful for large data sets. And they provide an idea of center, shape and spread and show unusual features of the data sets. However, individual data values are not included in histogram. **Note:** Bars should touch because the data is sequential/numerical in nature and both of the axes must be labeled.

Example: The following table provides a list of weights in kilograms for the forwards and backs of a rugby team that toured parts of New Zealand.

Backs	120	85	90	83	90	81	110	84	113	77
	75	87	100	94	91	79	100	82	94	102

Forwards	128	100	89	105	105	101	115	108	114	99
	98	104	108	101	100	99	110	105	105	115

Create a histogram for the Backs

Step 1: Input the data into L_1 and follow the calculator commands below:

- 2nd STAT PLOT
- ENTER
- Highlight ON
- ↓→highlight Histogram
- ↓Xlist
- 2nd "column number"
- Freq:1
- Zoom 9
- Window
- Xmin 60
- Xmax 140
- Xscl=10
- Graph & Trace

Step 2: Draw the graph

Step 3: Label the axis

Step 4: Title the graph

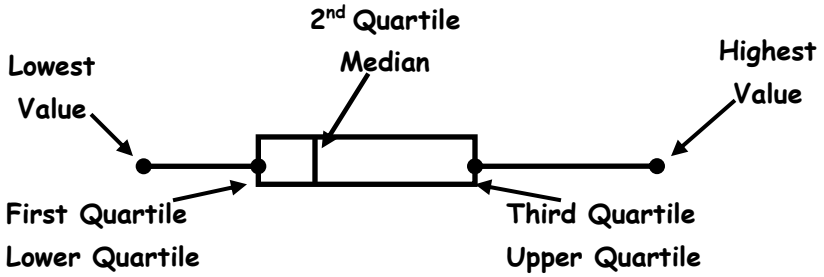
Input the data for the forwards into L_2 & Repeat the steps above. Compare and contrast the 2 groups.

Date:_____ Compiled by: Loren L. Spencer

Notes: Interpreting Boxplots

If I had to choose a data display that I though was the most important for AP Statistics, I would without hesitation say that it was the box and whisker plot (saved the best for last). From this point forward we will refer to the box and whisker plot as the box plot.

Box Plot: a visual representation of **the 5 number summary**. Each section of a boxplot contains 25% of the data. A boxplot can give some idea as to shape in particular symmetry and skewness. However exact shape of a distribution as well as, the mean and variance cannot be determined from a boxplot display.



Median: The data point that occurs in **the middle** when the numbers are placed in order from least to greatest. **(not impacted by outliers)**

Mean: The average of the data. **(greatly affected by outliers)**

Range: The difference between the highest and lowest value. **(Always affected by outliers)**

First Quartile: The median of the lower half of the data. 25% of the data are below the value.

Third Quartile (Q_3): The median of the upper half of the data. 75% of the data are below the value.

IQR: the interquartile range

$$IQR = Q_3 - Q_1$$

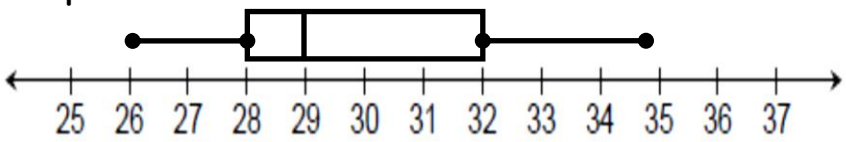
Outlier: A number that is too large or too small for compared to the rest of the data.

1st quartile−1.5×IQR

3rd quartile+1.5×IQR

Note: Box and whisker plots do not identify the mean.

Example:



a) Identify the highest value

b) Identify the lowest value

c) Identify the median

d) Find the range

e) Identify the first quartile

f) Identify the third quartile

g) Identify the interquartile range

h) 75% of the data are above what value?

i) 50% of the data are between which values? List all combinations.

j) Find the mean

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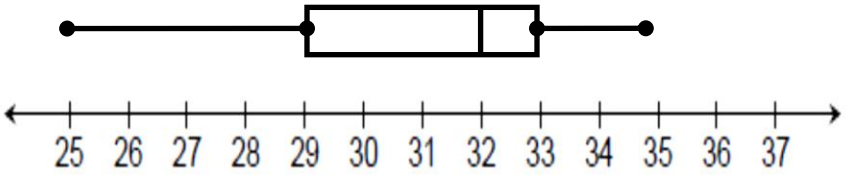


a) Identify the highest value

b) Identify the lowest value

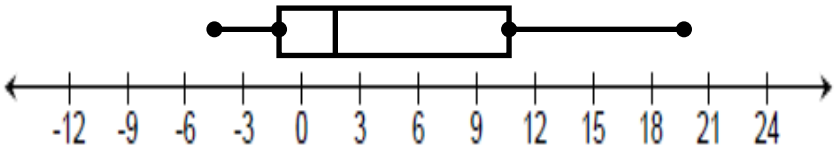
c) Identify the median

d) Find the range

e) Identify the IQR

f) How small or large would a value be to be considered an outlier?

g) Is the data skewed?



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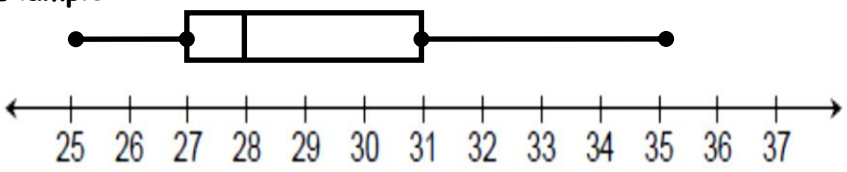

a) For which value of data is 50% of the values larger and 50% smaller?

b) What is the range of the data on the given graph?

Example: A box and whisker plot gives a picture of the test scores on a recent math test.

Using the box and whisker plot above answer the following questions:

a) True or false—We know that the class average was an 80.

b) True or false—We know that half of the students made less than an 80.

c) True or false—We know that the lowest grade was a 45.

d) True or false—We know that at least half of the class passed the test.

e) True or false—We know that the difference between the highest and lowest score was 55.

f) True or false—50% of the class made a B or better.

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Notes: Creating Boxplots

Box Plots

Median: The data point that occurs in **the middle** when the numbers are placed in order from least to greatest. **(not greatly affected by outliers)**

Creating Box Plots

Step 1: Press **STAT EDIT ENTER**
Input the data into column L_1

Step 2: Press **2nd STAT PLOT→ Enter→**(turn stat Plot on)
Press **↓→→→Enter**
Press **↓ 2nd L_1**
(Make certain Freq:1)

Step 3: Press **ZOOM 9**

5 Number Summary

Step 1: Press **STAT EDIT ENTER**
Input the data into column L_1

Step 2: Press **STAT→CALC ENTER**
(1-Var Stats)

Note: Make certain L_1 is selected

Step 3: Press **ENTER 3 times**

Step 4: Arrow down

- Min = smallest value
- Q_1 = 1st quartile
- Med = Median/2nd quartile
- Q_3 = 3rd quartile
- Max = largest value

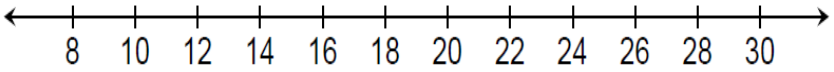
Note: Other information

- $\bar{x}$ = sample mean
- n = sample size
- s_x = sample standard deviation
- σ_x = population standard deviation

Box Plot: *A graphical display of data along a number line, divides the data into four parts called quartiles and identifying the lowest and highest data value along with the* **median**

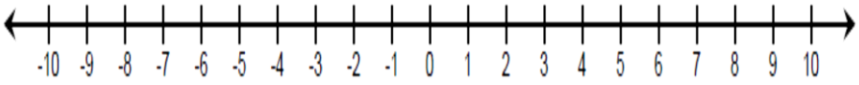
Example: Using the data below construct a box plot and show the construction of the fences for the outliers.

19, 17, 12, 24, 10, 19, 25, 15, 16, 15, 19, 16



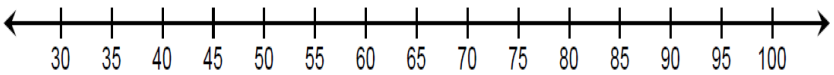
Example: Using the data below construct a box plot and show the construction of the fences for the outliers.

9, 8, 7, 6, 5, 4, 3, 2, 1, -10



Example: Using the data below construct a box plot and show the construction of the fences for the outliers.

80, 90, 60, 90, 85, 65, 95, 55, 70, 80, 100, 45, 95, 80, 60, 75, 80, 60, 50, 85



Notes: Displaying and Describing Data Scenarios

2000 Problem 3 Five hundred randomly selected middle-aged men and five hundred randomly selected young adult men were rated on a scale from 1 to 10 on their physical flexibility, with 10 being the most flexible. Their ratings appear in the frequency table below. For example, 17 middle-aged men had a flexibility rating of 1.

Physical Flexibility Rating	Frequency of Middle-Aged Men	Frequency of Young Adult Men
1	17	4
2	31	17
3	49	29
4	71	39
5	70	54
6	87	69
7	78	83
8	54	93
9	34	73
10	9	39

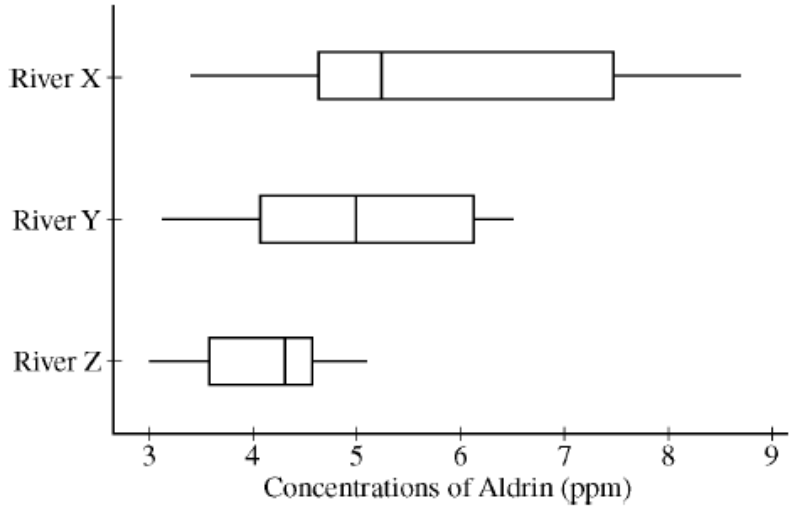
(a) Display these data graphically so that the flexibility of middle-aged men and young adult men can be easily compared.
Based on an examination of your graphical display, write a few sentences comparing the flexibility of middle-aged men with the flexibility of young adult men.

Notes: Displaying and Describing Data Scenarios

2010 Form B Problem 1 As a part of the United States Department of agriculture's Super Dump cleanup efforts in the early 1990s, various sites in the country were targeted for cleanup. Three of the targeted sites---River X,

River Y, and River Z---had become contaminated with pesticides because they were located near abandoned pesticide dump sites. Measurements of the concentration of aldrin (a commonly used pesticide) were taken at twenty randomly selected locations in each river near the dump sites.

the boxplots shown below display the five-number summaries for the concentrations, in parts per million (ppm) of aldrin, for the twenty locations that were sampled in each of the three rivers.



a) Compare the distributions of the concentration of aldrin among the three rivers.

b) The twenty concentrations of aldrin for River X are given below.

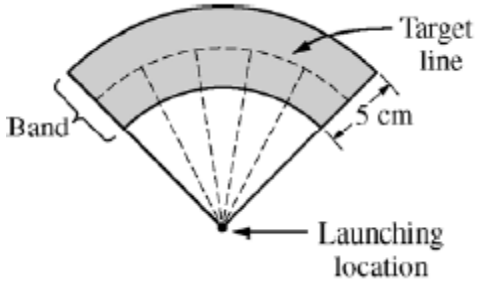
3.4	4.0	5.6	3.7	8.0	5.5	5.3	4.2	4.3	7.3
8.6	5.1	8.7	4.6	7.5	5.3	8.2	4.7	4.8	4.6

Construct a stemplot that displays the concentrations of aldrin for River X

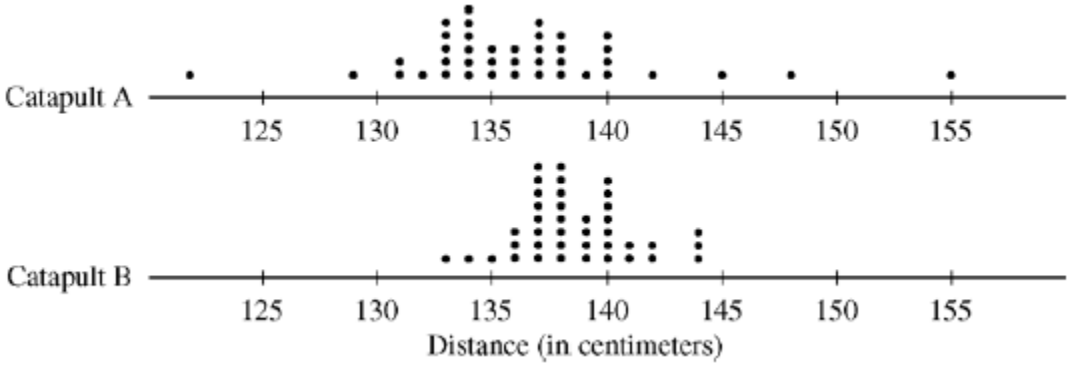
c) Describe a characteristic of the distribution of aldrin concentrations in River X that can be seen in the stemplot but cannot be seen in the boxplot.

Notes: Displaying and Describing Data Scenarios

2006 Problem 1: Two parents have each built a toy catapult for use in a game at an elementary school fair. To play the game, students will attempt to launch Ping-Pong balls from the catapults so that the balls land within a 5-centimere band. A target line will be drawn through the middle of the band, as shown in the figure below. All points on the target line are equidistant from the launching location.



If a ball lands within the shaded band, the student will win a prize. The parents have constructed the two catapults according to slightly different plans. They want to test these catapults before building additional ones. Under identical conditions, the parents launch 40 Ping-Pong balls from each catapult and measure the distance that the ball travels before landing. Distances to the nearest centimeter are graphed in the dotplots below.



(a) Comment on any similarities and any differences in the two distributions of distances traveled by balls launched from catapult A and catapult B.

(b) If the parents want to maximize the probability of having the Ping-Pong balls land within the band, which one of the two catapults, A or B, would be better to use than the other? Justify your choice.

(c) Using the catapult that you chose in part (b), how many centimeters from the target line should this catapult be placed? Explain why you chose this distance.

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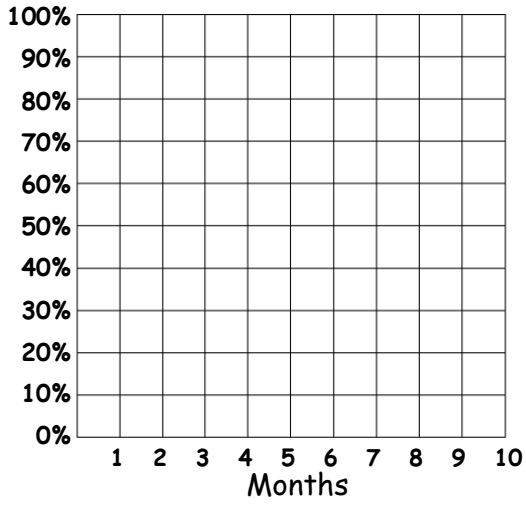
Notes: Cumulative Frequency Graphs

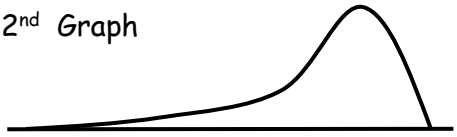
As one moves from left to right cumulative frequency graphs increase from zero percent until they reach 100%. Cumulative frequency charts never decrease.

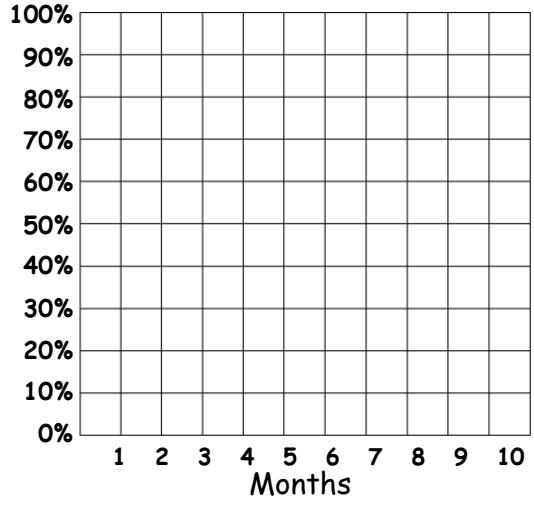
Scenario: The following graphs show the way 3 soccer clubs scored their points over a 10 month season.
1st Describe each of the shapes.
2nd Explain what each graph represents as far as to the distribution of goals throughout the season.
3rd Graph each as a cumulative frequency. Only focus on the shape of the graph

1st Graph

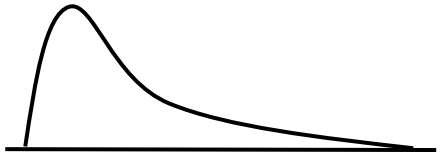


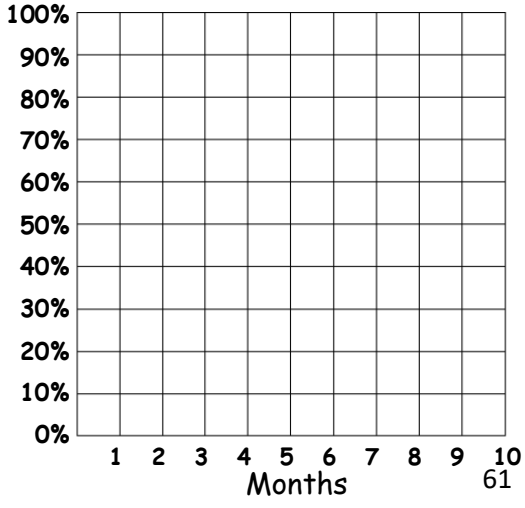
2nd Graph





3rd Graph



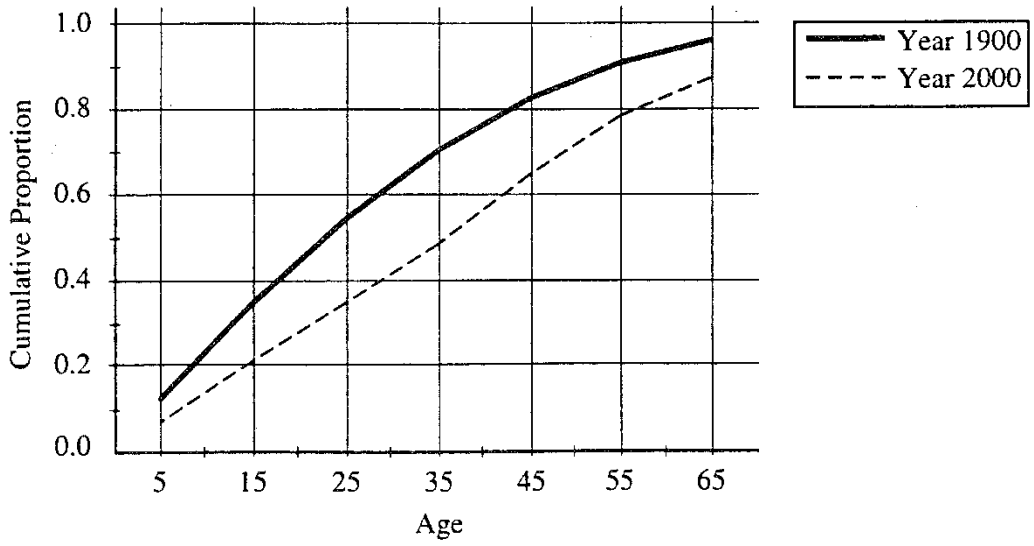


Notes: Cumulative Frequency Graph Scenarios

AGE DATA

Age	1900	2000
5	0.121	0.066
15	0.344	0.209
25	0.540	0.344
35	0.700	0.480
45	0.822	0.643
55	0.906	0.781
65	0.959	0.870

1999 Problem 1: The table of data above provides the cumulative proportions for the United States population at selected ages for the years 1900 and 2000 (projected). For example, 0.344 or 34.4 percent of the population was at or under age 15 in 1900, while only 0.209 or 20.9 percent will be at or under age 15 in the year 2000. The graph below shows the cumulative proportions plotted against age for the years 1900 and 2000 (projected). The data and graph are to be used to compare the age distribution for the year 1900 with the projected age distribution for the year 2000.



(a) Approximate the median age for each distribution.
(b) Approximate the interquartile range for each distribution.
(c) Using the results from parts (a) and (b), write a sentence or two for a history textbook comparing the age distributions for the years 1900 and 2000.

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Notes: Standardizing with Z-scores

Percentile Ranks-provide the percent of values that lie below a given value. A percentile rank tells us the location of the value in a distribution once the values are lined up smallest to largest

1. Here is a stemplot of the percents of residents aged 65 and older in the 50 States:

7	0
8	8
9	8
10	0 1 9
11	1 6 7 7 7
12	0 1 1 2 2 4 5 6 7 7 8 9 9 9
13	0 0 0 1 2 2 3 3 4 4 4 5 5 6 8 9
14	0 2 3 4 6 8
15	2 4
16	9

Key: 15|2 means 15.2% of this state's residents are 65 or older.

a) 10.1% of Colarado's residents are 65 or older. What is Colorado's percentile rank relative to the other states?

b) Rhode Island is at the 80th percentile of the distribution. Interpret this value. What percent of Rhode Island residents are 65 and older?

2. Many athletes worry about the risk of concussions when playing sports. A football coach plans to obtain specially designed helmets for his player that are designed to reduce the chance of getting a concussion. The table below lists the measurements of head circumference in caches for the players on the team.

23.0	**22.2**	**21.7**	**22.0**	**22.3**	**22.6**	**22.7**	**21.5**
22.7	**25.6**	**20.8**	**23.0**	**24.2**	**23.5**	**20.8**	**24.0**
22.7	**22.6**	**23.9**	**22.5**	**23.1**	**21.9**	**21.0**	**22.4**
23.5	**22.5**	**23.9**	**23.4**	**21.6**	**23.3**		

A) Raul, the team's starting quarterback, has a head circumference of 22.4 inches. What is Raul's percentile rank?

B) What is the head circumference for someone at the 20th percentile?

C) What is the head circumference for someone at the 80th percentile?

Notes: Standardizing with Z-scores

Z-scores: A z-score is a ratio that can be used to determine how many standard deviations a value lies from its mean by taking into account a measure of spread (the standard deviation of the distribution) and a measure of center (the mean of the distribution). A negative value lies below the mean and a positive value falls above the mean. A value at the mean, has a z-score of zero.

For individual values we use: $Z = \frac{x-\mu}{\sigma}$

For sample means we use: $Z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$ or $Z = \frac{\bar{x}-\mu}{\sigma_{\bar{x}}}$ where $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

Z-scores can be computed for any shaped distribution. Consequently, we can use z-scores as a ruler to compare scores from different distributions. We can say that this persons scored 1.2 standard deviations above the mean and this person scored 1.8 standard deviations above the mean and we can do that regardless of the underlying distribution and regardless of the sample size. Typically we will define values that are more than 3 standard deviations beyond the mean as outliers. In fact, we often say that a value that is more than 2 standard deviations beyond the mean is an outlier.

1. At a college the scores on the chemistry final exam are approximately normally distributed, with a mean of 75 and a standard deviation of 12. The scores on the calculus final are also approximately normally distributed, with a mean of 80 and a standard deviation of 8. A student scored 81 on the chemistry final and 84 on the calculus final. Relative to the students in each respective class, in which subject did this student do better?

What would a 60 on the Chemistry final equate to on the calculus final?

2. The table below describes the distribution of median household incomes in the 50 states.

n	Mean	SD	Min	Q_1	Med	Q_3	Max
50	51,742.44	8210.64	36,641,	46,071	50,009	57,020	71,836

a) Find and interpret the z-score for North Carolina with a median income of $41,553.

b) New Jersey has a standardized score of 1.82. Calculate New Jersey's median income.

Notes: Standardizing with Z-scores

2011 Question 1: A professional sports team evaluates potential players for a certain position based on two main characteristics, speed and strength.

Speed is measured by the time required to run a distance of 40 yards, with smaller times indicating more desirable (faster) speeds. From previous speed data for all players in this position, the times to run 40 yards have a mean of 4.60 seconds and a standard deviation of 0.15 seconds, with a minimum time of 4.40 seconds, as shown in the table below.

	Mean	Standard deviation	Minimum
Time to run 40 yards	4.60 seconds	0.15 seconds	4.40 seconds

Strength is measured by the amount of weight lifted, with more weight indicating more desirable (greater) strength. From previous strength data for all players in this position, the amount of weight lifted has a mean of 310 pounds and a standard deviation of 25 pounds, as shown in the table below.

	Mean	Standard deviation
Amount of weight lifted	310 pounds	25 pounds

(b) Calculate and interpret the z-score for a player in this position who can lift a weight of 370 pounds.

(c) The characteristics of speed and strength are considered to be of equal importance to the team in selecting a player for the position. Based on the information about the means and standard deviations of the speed and strength data for all players and the measurements listed in the table below for Players A and B, which player should the team select if the team can only select one of the two players? Justify your answer.

	Player A	Player B
Time to run 40 yards	4.42 seconds	4.57 seconds
Amount of weight lifted	370 pounds	375 pounds

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Notes: Calculating Means & Standard Deviations of Discrete Distributions

We previously calculated means, variance and standard deviations of data sets and used the following formulas: $\mu = \frac{\Sigma x_i}{n}$ **Mean** $\sigma^2 = \sum \frac{(x_i - \mu)^2}{n}$ **Variance** $\sigma = \sqrt{\sum \frac{(x_i - \mu)^2}{n}}$ **Standard deviation**

What if instead of having a list of data, we knew how often certain results occurred either by simulation of experience? For instance, we know that if we roll a die we the chance of getting a 1 is 1/6 and 2 is 1/6 etc. So the question is how do we calculate the mean variance and standard deviation?

Well first of all we need to understand that when we roll a die, we can only get a 1, 2, 3, 4, 5 or 6—duh. However that is important to not because those values 1, 2, 3, 4, 5, and 6, represent the entire population. Further we know the chance of any one of those values is 1/6. I have represented this complex data in the table below.

Population	1	2	3	4	5	6
Chance	1/6	1/6	1/6	1/6	1/6	1/6

Probability Distribution: A table or an equation that links each outcome of a statistical experiment with its probability of occurrence. To be a valid probability distribution the following must be true:

1. The sum of the probabilities must be 1.
2. The probabilities must lie between 0 and 1 inclusive.
   There is no such thing as a negative probability.
   There is no such thing as a probability greater than one.

Question: Does the table represent a valid probability distribution? Justify your response.

Now back to the original question: how do I calculate the mean, variance and standard deviation? Fortunately, we have a simple formula to do this and if you are curious I can easily demonstrate where this formula came from. Just ask.

E(x) = $\mu_x = \Sigma x_i p_i$

All we are doing is multiplying each outcome by its associated probability and then summing them.
In the space above calculate the mean.

Similarly the variance is found by squaring the difference between a mean and a value and then multiplying the result by the values probability the resulting products are then summed.

$\sigma^2 = \sum (x_i - \mu)^2 p_i$

In the space above calculate the variance.

The standard deviation is the square root of the variance: $\sigma = \sqrt{\sum (x_i - \mu)^2 p_i}$

Calculator Commands: Enter the outcomes in column L_1 the probabilities in column L_2.
Go to STAT CALC 1-VarStats. For List: enter L_1 for Frequency enter L_2.

Notes: Calculating Means & Standard Deviations of Discrete Distributions

Consider the following scenarios:

1. A commuter must pass through five traffic lights on her way to work. She estimates the probability model for the number of lights she hits as shown below. Calculate the expected value/mean and the the standard deviation and variance showing all work.

X	0	1	2	3	4	5
Probability	0.10	0.25	0.30	0.20	0.10	0.05

E(x) = σ = σ^2 =

---

a. Does the above scenario represent a valid probability distribution? Justify.

b. Calculate by hand the expected value and place in context of the problem:

E(x) = $\mu_x = \Sigma x_i p_i$

c. Calculate by hand the variance: $\sigma^2 = \Sigma (x_i - \mu_x)^2 p_i$

2. Complete the probability distribution. Calculate the expected value, the standard deviation and the variance for the following:

X	2	4	6	8
Probability	0.15	0.3	0.35	

E(x) = σ = σ^2 =

---

3. An insurance company charges$1,600 annually for car insurance. The policy specifies that the company will pay $3000 for a minor accident and $10,000 for a major accident. The probability of a motorist having a minor accident during the year is .22, and of having a major accident, .03. Complete the probability table below for how much money the insurance company will make on a single policy. **Hint:** Decide what X is.

X			
Probability			

E(x) = σ = σ^2 =

Notes: Transforming Random Variables

Adding Constants: Now that we know how to calculate the expected value and variance of a discrete random variable, what happens if we were to add a constant to every value in the distribution? Simply put nothing would happen to the measures of spread. The standard deviation, variance, range and IQR would remain unchanged. However the mean and median would change by the value of the constant.

Consider the following scenario: On the last test there were the following scores: 100, 73, 83, 100, 83, 92, 92, 28, 88, 68, 39, 92, 100, 92, 83, 78, 83, 92, 28, 96

Input the data using **STAT Edit** and calculate using **STAT Calc 1-var stats**

Mean Median

Standard deviation Variance

Range IQR

As your teacher I decided to add 5 points to each grade. Recalculate the statistics:

Hint: Highlight L_2 and press 2^{nd} 1 + 5 and calculate using Stat Calc 1-var stats

Mean Median

Standard deviation Variance

Range IQR

What happened to the measures of spread?

What happened to the measures of center?

Multiplying by Constants: Multiplying every test score by a constant has the same effect as multiplying the measures of center by the constant. The same is true with the measures of spread with the exception of variance. Variance is the square of the standard deviation, thus variance is multiplied by the square of the constant.

Consider the above scenario: Adjust the original test scores by using the AP multiplier of 1.29

Mean Median

Standard deviation Variance

Range IQR

What happened to measures of center, spread and variance?

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Notes: Combining Random Variables

So what happens if we combine 2 random variables?

Expected Values: It turns out that the mean or expected values of random variables can be added or subtracted directly and no conditions apply.
(note: if we combine 2 normal distributions, the result is a normal distribution**)**

Variances: Unlike expected values, there are limitations with the combining of random variables when it comes to measures of spread. **First:** we can only combine variances. **Second:** we can only combine variance is if the **random variables are independent**. **Third:** we can only add the variance. This means that we sum the variances when we are asked to sum them and we sum them when we are asked to subtract them.

Note: If we wish to combine standard deviations, we must do the following: square the standard deviation; follow the rules for variances; take the square root of the resulting variance to find the standard deviation.

Example: Given independent random variables with means and standard deviations as shown, find the mean and standard deviation of each of these variables:

	Mean	Standard Deviation
X	10	2
Y	20	5

(a) **3X**

(b) **Y+6**

(c) **X+Y**

(d) **X-Y**

(e) $\mathbf{X_1+X_2}$

(f) **X÷2 + Y**

(g) **2Y + 20**

Notes: Combining Random Variables Scenario

Scenario: A small business just leased a new computer and color laser printer for three years. The service contract for the computer offers unlimited repairs for a fee of $100 a year plus a $25 service charge for each repair needed. The company's research suggested that during a given year 86% of these computers needed no repairs, 9% needed to be repaired once, 4% twice, 1% three times, and none required more than three repairs.

1. Find the expected number of repairs this kind of computer is expected to need each year. Show your work.

2. Find the standard deviation of the number of repairs each year. Show your work.

3. What are the mean and standard deviation of the company's annual expense for the service contract?

4. How many times should the company expect to have to get this computer repaired over the three-year term of the lease?

5. What is the standard deviation of the number of repairs that may be required during the three-year lease period? On what assumption does your calculation rest? Do you think this assumption is reasonable? Explain.

6. The service contract for the printer estimates a mean annual cost of $120 with standard deviation of $30. What is the expected value and standard deviation of the total annual cost for the service contracts on computer and printer?

7. Which service contract should the company expect to cost more each year? How much more? With what standard deviation?

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Notes: Bivariate Data- A Very Brief Introduction

Bi-variate Data, as you may have surmised by the name, bi-variate data is two variable data. (Typically the variables are related or we believe them to be.) We have already spent a good deal of time dealing with bivariate data that is categorical in nature. (recall the two way tables). We have not addressed Bi-variate data that is quantitative in nature. **We will be discussing bi-variate data in great detail during the spring semester when we study regression.** I am only mentioning it here because you need to know that we are not skipping it. We are only postponing it.

You will be accountable for the information below.

Examples of Quantitative bivariate data: strength and age; electricity generation and hours of sunlight; blood pressure and weight. All of these have numerical measurements and are more than just counts.

In Algebra 1, you dealt with quantitative bi-variate data extensively when you dealt with linear equations. Consider the slope intercept form of the line **y = mx + b**. X and Y are variables and m provides the relationship between those variables.

As you may recall, a great deal of time was spent finding the equation of a line from a graph. This is very similar to what we will be doing in this class. There is one primary difference. In Algebra, the points all fell on the line. In statistics, we learned that data is not perfect and thus it is highly unlikely that all of the data points will lie on the line. Fitting a line to the data is known as regression and the line is called the least squares regression line because our goal is to minimize the distances the line is from the actual data. The close the line to the actual data the better the fit.

You need to know that the graph of the line that contains the points of interest is known as a **scatterplot**. The y axis records the responses and the x axis provides the inputs. The plot demonstrates the relationship between the two variables.

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Notes: Introduction to Probability

Probability: The ratio comparing what you want to happen to the total number of possible outcomes. How likely something is to happen. **ALL** probabilities will range from 0 to 1 inclusive.

$$\textbf{Probability} = \frac{\textbf{Number of favorable outcomes}}{\textbf{Number of total possible outcomes}} \quad \textit{or} \quad \frac{\textbf{What you want to happen}}{\textbf{Things that could happen}}$$

- The probability of the impossible is 0 or 0%
- The probability of something guaranteed to happen is 1 or 100%

Random phenomenon-possible outcomes are known, but the outcome for a particular event is unknown.

Sample Space is the list of all possible outcomes

Trial-a single event ex. 1 roll of a dice, 1 spin of a spinner, 1 flip of a coin, 1 guess on a test.

Law of Large Numbers-as the number of trials increases the percentage of successes moves closer to the expected number of successes-the theoretical number of successes. This occurs in the long run or repeated trials. **Does Not apply to individual trials or events.**
Remember: The past does not affect the future if the events are independent.

Independent events: Events in which the probability that one event occurs in no way affects the probability of the other event occurring.
Multiplication Principal-if two events are independent their probabilities can be multiplied to find the likelihood that both events will occur at the same time.
Equation $P(A \cap B) = P(A) \times P(B)$ this reads the probability of the intersection of A and B, also A and B, is equal to the probability of A times the probability of B.

DO NOT USE THIS UNLESS THE EVENTS ARE INDEPENDENT!!!

Hint: **"Or"** means **Add** & **"And"** means **Multiply** when doing probability.

Example: A fair number cube is rolled and a fair coin is tossed.

a) What is the probability of rolling a 3 and getting a tails?

b) What is the probability of rolling two 4's in a row followed by tossing 2 heads in a row

c) What is the probability of rolling a 5 or 6 and then tossing 3 heads in a row?

Example: A bag contains 3 A's and 5 B's.

a) What is the probability of drawing an A, rolling a 6 on the dice and tossing a tail on the coin?

b) What is the probability of drawing a B and getting a heads on the coin and either a 2 or 3 on the dice?

Notes: Basic Probability Scenario

Scenario 1: Five multiple choice questions, each with four possible answers, appear on your history exam. What is the probability that if you just guess, you

a. get none of the questions correct?

b. get all of the questions correct?

c. get at least one of the questions wrong?

d. get your first incorrect answer on the fourth question?

Scenario 2: The Masterfoods company manufactures bags of Peanut Butter M&M's. They report that they make 10% each brown and red candies, and 20% each yellow, blue, and orange candies. The rest of the candies are green.

a. If you pick a Peanut Butter M&M at random, what is the probability that

i. it is green?

ii. it is a primary color (red, yellow, or blue)?

iii. it is not orange?

b. If you pick four M&M's in a row, what is the probability that

i. they are all blue?

ii. none are green?

iii. at least one is red?

iv. the fourth one is the first one that is brown?

c. After picking 10 M&M's in a row, you still have not picked a red one. A friend says that you should have a better chance of getting a red candy on your next pick since you have yet to see one. Comment on your friend's statement.

Notes: Introduction to Probability

Probability: The ratio comparing what you want to happen to the total number of possible outcomes. How likely something is to happen. **ALL** probabilities will range from 0 to 1 inclusive.

- The probability of the impossible is 0 or 0%
- The probability of something guaranteed to happen is 1 or 100%

Dependent events: A group of simple events where one outcome **does** affect the other.

Example: A bag contains 3 A's and 5 B's

a) What is the probability of drawing two A's at the same time?

b) What is the probability of drawing a B on the first draw and then drawing an A without replacement?

c) What is the probability of drawing 3 A's in a row without replacement?

d) What is the probability of drawing 2 A's in a row and then drawing a B without replacement?

e) What is the probability of drawing an A replacing the letter and then drawing a B?

f) What is the probability of drawing a B replacing the letter and then drawing an A?

Complementary Event- the probability that an event will not occur. The probability of the complement is equal to 1 minus the probability that the event will occur.

Equation $P(A^C)=1-P(A)$ this reads the probability of the compliment A occurring is equal to 1- Probability of "A" occurring. A^C is similar to NOT "A".

a) What is the probability of not rolling a 3 on a six sided dice?

b) What is the probability not drawing an ace from a standard deck of cards?

c) If I draw 3 cards without replacement what is the probability that none are aces?

At Least Questions: If you have **at least** questions you must use the complement

Example: If a student guesses on a 5 question multiple choice quiz with 4 answer choices what it the probability that the student gets at least 1 wrong.

Example: A machine has 4 components the probability that the components do not fail during the year are: .97, .99, 98 and .96. If any component fails the machine fails, what is the probability that it fails?

Notes: Complement Scenarios

Scenario: 2004 Question 4: Two antibiotics are available as treatment for a common ear infection in children.

- Antibiotic A is known to effectively cure the infection 60 percent of the time. Treatment with antibiotic A costs $50.
- Antibiotic B is known to effectively cure the infection 90 percent of the time. Treatment with antibiotic B costs $80.

The antibiotics work independently of one another. Both antibiotics can be safely administered to children. A health insurance company intends to recommend one of the following plans for treatment for children with this ear infection.

- Plan I: Treat with antibiotic A first. If it is not effective, then treat with antibiotic B.
- Plan II: Treat with antibiotic B first. If it is not effective, then treat with antibiotic A.

(a) If a doctor treats a child with an ear infection using plan I, what is the probability that the child with be cured?
If a If a doctor treats a child with an ear infection using plan II, what is the probability that the child with be cured?

(b) Compute the expected cost per child when plan I is used for treatment.
Compute the expected cost per child when plan II is used for treatment.

(c) Based on the results in parts (a) and (b), which plan would you recommend? Explain your recommendation.

Notes: Sample Space

On any probability problem it is always a good idea to create a sample space. A sample space is a list of all possible outcomes along with their frequencies. This is especially true of games or events that are played twice. The classic example of this are dice questions

Example: What is the probability of rolling a fair six sided dice twice and getting sum less than 8? Create a sample space to answer the question.

Given that the sum is less than 8, what is the probability that the sum was equal to 5?

Given that the sum is less than 8, what is the probability that the sum was greater than 4?

Given that the sum is less than 8, what is the probability that the sum is 3 or less?

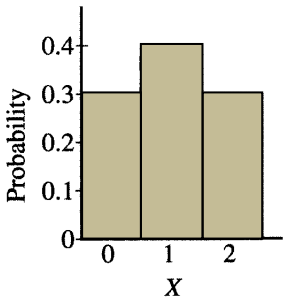
What if the die had been weighted and the probabilities were as follows:

Number Shown	1	2	3	4	5	6
Probability	.10	.10	.10	.20	.20	.30

Calculate the probability of Doubles

Calculate the Probability that the sum equals 6

Example: A game of chance is played in which X, the number of points scored in each game, has the distribution shown at the right. What is the sampling distribution of the sum, Y, of the scores when the game is played twice?



Game 1 Score Distribution of (X)

Game 2 Score distribution of (X)			
	(0, 0)	(1, 0)	(2, 0)
	(0, 1)	(1, 1)	(2, 1)
	(0, 2)	(1, 2)	(2, 2)

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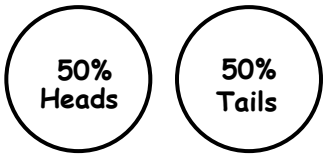
Notes: Classical Probability & Venn Diagrams

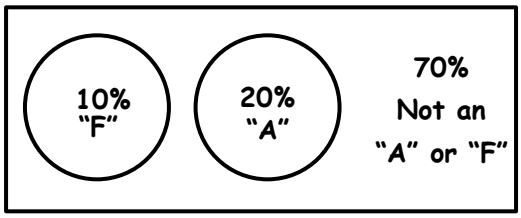
Mutually Exclusive Events/Disjoint Events-Events that cannot occur at the same time.

Mutually Exclusive is the same as Disjoint

Example: It is impossible to flip one coin and get both a heads and a tails on the same flip.



Example: You cannot make a 100% on the test and make an F on the test at the same time. Let's say the probability of an A is 20% the probability of an F is 10% and the probability of everything else sums to 70%.



Complementary Event- the probability that an event will not occur. The probability of the complement is equal to 1 minus the probability that the event will occur.

Equation $P(A^C)=1-P(A)$ this reads the probability of the compliment A occurring is equal to 1- Probability of "A" occurring. A^C is similar to NOT "A".

Refer to the Examples Above. Give the solutions and the associated Probabilities for each

The complement of heads is **The complement of "A" is**

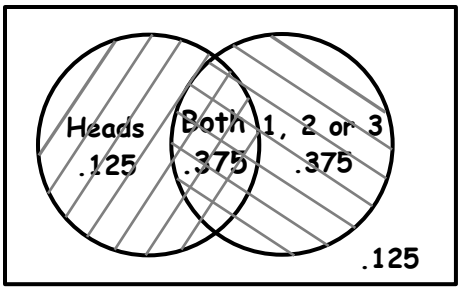
The complement of tails is **The complement of "F" is**

Independent Events-Events that do not rely on one another. The outcome of one does not impact the outcome of the other. **If independent $P(A\cap B) = P(A) \times P(B)$**

***Mutually Exclusive* is not the same as Independent**

Note: Events **cannot** be mutually exclusive & Independent unless one event's probability is **ZERO**.

Example: A game requires that you roll a fair 4 sided die numbered 1 thru 4 and to flip a coin. The Venn Diagram illustrates the probability of getting a heads and rolling a 1, 2 or 3. Where did those probabilities come from?



General Addition Rule-may be used anytime 2 probabilities are being summed but is necessary when 2 events that are not mutually exclusive are being summed. **Equation** $P(A\cup B) = P(A) + P(B) - P(A\cap B)$ **this reads (A union B), *also A or B*,** is equal to Probability of A plus the probability of B minus the probability of the intersection of (A and B)

Note: $P(A\cap B)$ *is the probability of both events occurring at the same time* ***it is the overlap****. There is not an overlap if the events are disjoint.*

Refer to the Game Example: Use the formula calculate the probability of getting a 1, 2, or 3 or a head.

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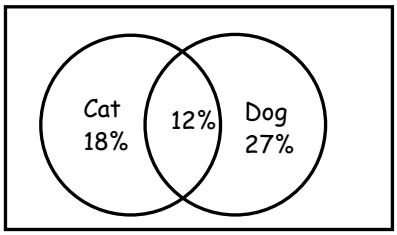
Notes: Classical Probability & Venn Diagrams

Example: The Venn diagram illustrates the probability that a family has a cat or dog or both.



What is the probability of having both a Cat and a Dog?

What is the probability of having a Cat?

What is the probability of having a dog?

What is the Probability of not having a Cat?

What is the Probability of not having a Dog?

What is the Probability of not having both a Cat and a Dog?

What is the Probability of having neither a cat nor a dog?

Are having a dog or cat independent events? Justify your answer.

What is the probability of having a cat or a dog? ***Use the formula:***

What is the probability of having only a dog? What would probability of only a dog need to be for cat and dog to be independent events?

Complete the Contingency table for the above situation.

	Dog	**No Dog**	**Total**
Cat			
No Cat			
Total			

Given someone has a cat, what is the probability that they have a dog?

Given that a person has a dog, what is the probability that they have a cat?

Notes: Venn Diagram Scenarios

Recall everyone's favorite antibiotic scenario.

Scenario: 2004 Question 4: Two antibiotics are available as treatment for a common ear infection in children.

- Antibiotic A is known to effectively cure the infection 60 percent of the time. Treatment with antibiotic A costs $50.
- Antibiotic B is known to effectively cure the infection 90 percent of the time. Treatment with antibiotic B costs $80.

The antibiotics work independently of one another. Both antibiotics can be safely administered to children. A health insurance company intends to recommend one of the following plans for treatment for children with this ear infection.

- Plan I: Treat with antibiotic A first. If it is not effective, then treat with antibiotic B.
- Plan II: Treat with antibiotic B first. If it is not effective, then treat with antibiotic A.

(a) If a doctor treats a child with an ear infection using plan I, what is the probability that the child with be cured?

If a If a doctor treats a child with an ear infection using plan II, what is the probability that the child with be cured?

(b) Compute the expected cost per child when plan I is used for treatment.
Compute the expected cost per child when plan II is used for treatment.

(c) Based on the results in parts (a) and (b), which plan would you recommend? Explain your recommendation.

Solve using a Venn Diagram:

Hint: the events are independent, begin with the overlap

Or

Notes: 2-Way Tables & Conditional Probability

A 2-way table is way to organize the probabilities of two variables. The outside row and column totals are the **marginal** values and represent the probability of a variable' response out of the total population. The inside values of the table are **conditional** values and demonstrate the probability that the interaction of the 2 variables would result in a particular combination or intersection of responses.

Consider the following Scenario:
Suppose that 2% of a clinic's patients are known to have cancer. A blood test is developed that is positive in 98% of patients with cancer but is also positive in 3% of patients who do not have cancer.

Step 1: Identify the 2 variables
Step 2: Identify the one probability that is related to only one of the two variables and record its value at the bottom of the 1st column and fill in the remaining column.

Cancer Status

Test Results	Cancer	No Cancer	Total
Positive			
Negative			
Total			1.0

Step 3: Label each variables potential value
Step 4: Using small writing, record the marginal probability of the columns into the conditional cells.
Step 5: Using small writing, record the remaining probabilities that are associated with the 2-variables.
Step 6: Multiply the values in the interior cells to find the conditional probabilities
Step 7: Sum each row's conditional probabilities to find the Marginal probabilities for each row.

a) What is the probability that the test is positive?

b) What is the probability that the test is negative and the person has cancer?

Given the person has cancer, what is the probability that the test is negative?

d) If a person who is chosen at random from the clinic's patients is given the test and it comes out positive, what is the probability that the person actually has cancer?

Notes: 2-Way Tables & Conditional Probability

Example: Leah is flying from Boston to Denver with a connection in Chicago. The probability her first flight leaves on time is 0.15. If the flight is on time, the probability that her luggage will make the connecting flight in Chicago is 0.95, but if the flight is delayed, the probability that the luggage will make it is only 0.65.

(a) What is the probability that he luggage arrives in Denver with her?

Make a 2-way table for the questions above.

	Flight On time	Flight Delayed	**Total**
Luggage With her	**.1425**	**.5525**	**.695**
Luggage Delayed	**.0075**	**.2975**	**.305**
Total	**.15**	**.85**	**1.0**

Draw a tree-diagram to answer the questions below

(b) Are the first flight leaving on time and the luggage making the connecting flight independent events? Explain.

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Notes: Conditional Probability & Contingency Tables

Conditional Probability- *Gives* the probability that event will happen given that another event has already happened. This is the same as a joint probability.

Equation $P(A|B) = \frac{P(A \cap B)}{P(B)}$ **or** $\frac{conditional}{marginal}$ **or** $\frac{joint}{marginal}$ **or** $\frac{and/intersection}{given}$

- *this means the probability of A given that B has occurred is equal to the probability of both events occurring divided by the probability of event B occurring.*

Example: The table gives the results of a survey of the drinking and smoking habits of 1200 college students. Rows and columns have also been summed.

	Drinks	Doesn't Drink	Total
Smoke	**315**	**165**	
Doesn't Smoke	**585**	**135**	
Total			

A) What is the probability that a student smokes?

B) What is the probability that a student drinks?

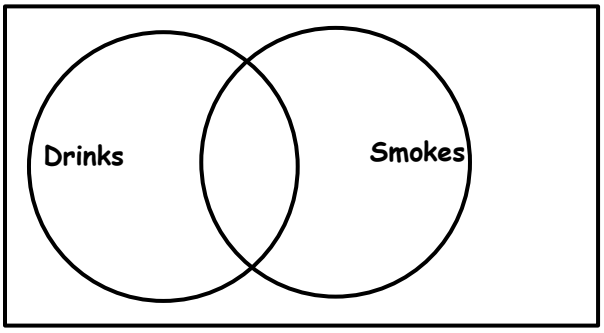
C) What is the probability that a student smokes and drinks?

D) What is the probability a student smokes given that the student drinks?

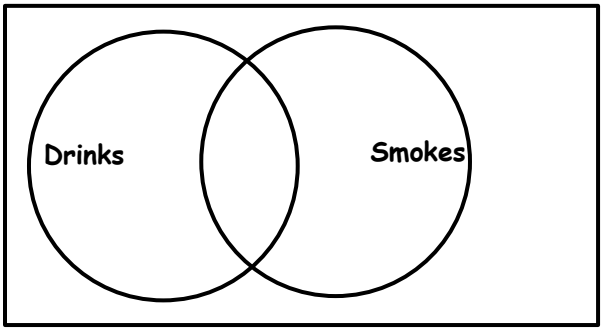
E) What is the probability that a student drinks given that the student smokes?

F) Why are the probabilities for **D** and **E** different?

Create a Venn Diagram using the actual values as opposed to the probabilities



Create a Venn Diagram using the probabilities as opposed to the values



Notes: Conditional Probability & Independence

Conditional Probability & Independence-if **P(A | B) = P(A)** then the events are independent

Or if $\frac{\mathbf{P(A\cap B)}}{P(B)}$ **= P(A) then independent. *Also if*** $\frac{\mathbf{P(A\cap B)}}{P(A)}$ **= P(B) then independent**

Example: Recall the results of right-handedness and left handedness as relates to I.Q. scores.

	Right handed	Left Handed	Total
High IQ	190	10	200
Normal IQ	1,710	90	1800
Total	1900	100	2000

	Right handed	Left Handed	Total
High IQ	.095	.005	.10
Normal IQ	.855	.045	.90
Total	.95	.05	1.0

Previously we used proportionality to determine that hand dominance and IQ were independent.

Demonstrate Independence using the multiplication rule:

- Using Percentages (right table)

- Using Actual Values (left table)

Demonstrate Independence using Conditional Probability:

- Using Percentages (right table)

- Using Actual Values (left table)

Notes: Conditional Probability & Contingency Tables Scenario

2003 Form B Problem 2: A simple random sample of adults living in a suburb of a large city was selected. The age and annual income of each adult in the sample were recorded. The resulting data are summarized in the table below.

	Annual Income			
Age Category	$25,000-	$35,001-$50,000	Over $50,000	Total
21-30	8	15	27	50
31-45	22	32	35	89
45-60	12	14	27	53
Over 60	5	3	7	15
Total	47	64	96	207

(a) What is the probability that a person chosen at random from those in this sample will be in the 31-45 paid category?

(b) What is the probability that a person chosen at random from those in this sample whose incomes are over $50,000 will be in the 31-45 age category? Show your work.

(c) Based on your answers to parts (a) and (b), is annual income independent of age category for those in this sample? Explain.

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Notes: Geometric Distributions

Bernoulli Trials- a random experiment with exactly two possible outcomes, "success" and "failure", in which the probability of success is the same every time the experiment is conducted.

Characteristics:

1. two possible outcomes (success and failure).
2. the probability of success, *p*, is constant.
3. the trials are independent

Note: When we don't have an infinite population, the trials are not independent. However it is still okay to proceed as long as the sample is smaller than 10% of the population. **M&M example.**

Geometric Distribution-has only 2 possible outcomes {success (p) and failure (q) q=1-p} Each trial is independent, the probabilities cannot change & the **number of trials not known.**

Equation: $q^{k-1}p$ **k is the number of trials until the 1st success**

Mean or E(X) $\mu = \frac{1}{p}$ **Variance or Var(X)** $\sigma^2 = \frac{q}{p^2}$ **Standard deviation** $\sigma = \frac{\sqrt{q}}{p}$

Geometrics are usually phrased as:

- What is the probability that the **1st** success will occur on a given trial (Geometric PDF)
- What is the probability that the **1st** success will occur no later than or by (Geometric CDF)

Example: Suppose only 17% of guys at some high school are honest. What is the probability that the probability that the 1st honest guy that a new girl at school meets will be the 5th guy to whom she is introduced?

Calculator: 2nd VARS ↓ E:geometpdf(

What is the probability that that the **first** honest guy she encounters will be no later than the 4th guy she meets?

Calculator: 2nd VARS ↓ F:geometcdf(

How many guys does she expect to have to meet to find the first honest guy?

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Notes: Geometric Distribution Scenarios

Example: The probability that any terminal is ready to transmit is .95. How many terminals will need to be tested until we find one that is ready to transmit?

What is the probability that the first terminal ready to transmit is the 1st terminal tested?

What is the probability that the first terminal ready to transmit is the 3rd terminal tested?

What is the expected number of terminals that will need to be tested until we find one not ready to transmit?

What is the probability that the first terminal not ready to transmit is the 10th terminal tested?

What is the probability that the first terminal not ready to transmit is the 20th terminal tested?

What is the probability that the first terminal not ready to transmit is the 30th terminal tested?

What is the likelihood that the first terminal not ready to transmit is found before the 100th terminal is checked?

Example: 2% of cars have a defective seatbelt. How many cars would you expect to have to check to find one with a defective seatbelt?

What is the probability that the first defective seatbelt is the 50th seatbelt checked?

What is the probability that the first defective seatbelt is found prior to the 51st seatbelt being checked?

Notes: Combinations

Factorial: The product of an integer and all integers below it.

Example: 5 Factorial is written 5! = 5×4×3×2×1; 5 Factorial is written 8! = 8×7×6×5×4×3×2×1

Note: 0! is equal to 1

Combination: a subset or selection of a group in which the order does not matter. It is often expressed as n choose k and written n(C)k which means n combination k.
We most often will use the notation: $\binom{n}{k}$

The Formula for $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ tells us **k** the number of different subsets that can be taken from a group of size **n**

- n is the size of the entire group;
- **k** is the size of the selection from the group

How many different subsets of 3 can be chosen from a group of 5? $\binom{5}{3}$

$$\frac{n!}{k!(n-k)!} = \frac{5!}{3!(5-3)!} = \frac{4!}{3!(2)!} = \frac{5\times4\times3\times2\times1}{3\times2\times1(2\times1)} = \frac{120}{6(2)} = \frac{120}{12} = 10$$

Consider the following: You have 4 people Aaron, Bob, Corina and Daniella that we call A, B, C, & D that we are going to make groups with.

1st group is to be 4 people: $\binom{4}{4} = \frac{4!}{4!(4-4)!} = \frac{4!}{4!(0)!} = \frac{4\times3\times2\times1}{4\times3\times2\times1(1)} = \frac{24}{24(1)} = \frac{24}{24} = 1$

Remember order doesn't matter, so there is only one way to have a group of 4 selected from a group of 4 and that is to choose all 4 Aaron, Bob, Corina and Daniella.

2nd group is to be 3 people: $\binom{4}{3} = \frac{4!}{3!(4-3)!} = \frac{4!}{3!(1)!} = \frac{4\times3\times2\times1}{3\times2\times1(1)} = \frac{24}{6(1)} = \frac{24}{6} = 4$

Using the letters we could have the following groups:

A,B,C; B,C,D; C,D,A; or **D,A,B**

3rd group is to be 2 people: $\binom{4}{2} = \frac{4!}{2!(4-2)!} = \frac{4!}{2!(2)!} = \frac{4\times3\times2\times1}{2\times1(2\times1)} = \frac{24}{2(2)} = \frac{24}{4} = 6$

Using the letters we could have the following groups:

A,B; A,C; A,D; B,C; BD; or **C,D;**

4th group is to be 1 person: $\binom{4}{1} = \frac{4!}{1!(4-1)!} = \frac{4!}{1!(3)!} = \frac{4\times3\times2\times1}{1(3\times2\times1)} = \frac{24}{1(6)} = \frac{24}{6} = 4$

Using the letters we could have the following groups:

A; B; C; or **D**

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Notes: Combinations

The Formula for $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ tells us **k** the number of different subsets that can be taken from a group of size **n**

- n is the size of the entire group;
- **k** is the size of the selection from the group

Example: You are making a sandwich. How many different combinations of 2 ingredients can you make with cheese, mayo and ham? **3 choose 2**

Answer: *{cheese, mayo}, {cheese, ham} or {mayo, ham}*

How many combinations of all 3 ingredients are there? **3 choose 3**

Try the following:

- An election ballot asks voters to select three city commissioners from a group of six candidates. In how many ways can this be done?

- A four-person committee is to be elected from an organization's membership of 11 people. How many different committees are possible?

- You are on your way to Hawaii (Aloha) and of 15 possible books your parents say you can only take 10. How many different collections of 10 books can you take?

- There are 12 standbys who hope to get on your flight to Hawaii, but only 6 seats are available on the plane. How many different ways can the 6 people be selected?

- To win the small county lottery, one must correctly select 3 numbers from 30 numbers. The order in which the selection is made does not matter. How many different selections are possible?

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Notes: Binomial Distribution

Bernoulli Trials- a random experiment with exactly two possible outcomes, "success" and "failure", in which the probability of success is the same every time the experiment is conducted.

Characteristics:

1. two possible outcomes (success and failure).
2. the probability of success, *p*, is constant.
3. the trials are independent

Note: When we don't have an infinite population, the trials are not independent. However it is still okay to proceed as long as the sample is smaller than 10% of the population. **M&M example.**

Binomial Distribution-has only 2 possible outcomes {**success (p) and failure (q) q=1-p**} Each trial is independent, the probabilities cannot change & the number of trials is pre-determined/fixed.

1. two possible outcomes (success and failure).
2. the probability of success, ***p,*** is constant as is the probability of failure ***q*** which is **1-p**
3. the trials are independent
4. **a specified number of trials will be completed whether there is one success, no successes or multiple successes**
5. Number of trials, successes and failures are discrete

Equation: $\sum_{k=0}^{n} \binom{n}{k} p^k q^{n-k} = \frac{n!}{k!(n-k)!} p^k q^{n-k}$

- **n is the number of trials**
- **k is the number of successes**

Mean or E(X)=np **Variance or Var(X)** σ^2 **= npq** **Standard deviation** $\sigma = \sqrt{npq}$

Binomials are usually phrased as:

- What is the probability of some number of successes in a given number of trials? (Binomial PDF)

- What is the probability of at least some number of successes in a given number of trials? (Binomial CDF) usually **1-**Binomial CDF

- What is the probability of no more than some number of successes in a given number of trials? (Binomial CDF)

- What is the probability that the number of successes in a given number of trials are between 2 values? (Binomial CDF)--**(Binomial CDF of larger value) - (Binomial CDF smaller value)**

Notes: Binomial Distribution Scenarios

Equation: $\sum_{k=0}^{n}\binom{n}{k}p^k q^{n-k} = \frac{n!}{k!(n-k)!}p^k q^{n-k}$

Mean or E(X)=np **Variance or Var(X)** σ^2 **= npq** **Standard deviation** $\sigma = \sqrt{npq}$

Binomials are usually phrased as:

- **(Binomial PDF):** What is the probability of an exact number of successes for a pre-determined number of trials?

  Calculator Commands

  Press 2nd VARS
  ↓A: binompdf(
  Press ENTER
  trials: Enter the number of trials. **Press ENTER**
  p: Enter the probability of success. **Press ENTER**
  x value: Enter the **exact** number of successes. **Press ENTER 3 times**

Examples:

Suppose that the probability of having a defective item is .3. Given a box of 5 light bulbs:

- What is the probability of having 0 defective items?
- What is the probability of having 1 defective items?
- What is the probability of having 2 defective items?
- What is the probability of having 3 defective items?
- What is the probability of having 4 defective items?
- What is the probability of having 5 defective items?
- What is the standard deviation and expected number of defects in a box of 5 items?

Notes: Binomial Distribution Scenarios

Equation: $\sum_{k=0}^{n}\binom{n}{k}p^k q^{n-k} = \frac{n!}{k!(n-k)!}p^k q^{n-k}$

Mean or E(X)=np **Variance or Var(X)** σ^2 **= npq** **Standard deviation** $\sigma = \sqrt{npq}$

- **(Binomial CDF):** What is the probability of **no more** than some number of successes for a pre-determined number of trials?-**no more, no greater than, less than or equal to, and the special case less than.**

  Calculator Commands

  Press 2nd VARS
  ↓B: binomcdf(
  Press ENTER
  trials: Enter the number of trials. **Press ENTER**
  p: Enter the probability of success. **Press ENTER**
  x value: Enter the **maximum** number of successes. **Press ENTER 3 times**

Examples:

- What is the probability of having no more than 3 defective items in a box of 5?

- What is the probability of having at least 3 defective items in a box of 5?

- What is the probability of having 2 to 4 defective items in a box of 5?

Notes: Binomial Distribution Scenarios

Binomial Equation: $\sum_{k=0}^{n}\binom{n}{k}p^k q^{n-k} = \frac{n!}{k!(n-k)!}p^k q^{n-k}$

n is the number of trials, **k** is the number of successes and **q** is the number of failures

Binomials are usually phrased as:

- **(Binomial PDF):** What is the probability of an exact number of successes for a pre-determined number of trials?

  Calculator Commands

  Press 2nd VARS
  ↓A: binompdf(
  Press ENTER
  trials: Enter the number of trials. **Press ENTER**
  p: Enter the probability of success. **Press ENTER**
  x value: Enter the **exact** number of successes. **Press ENTER 3 times**

- **(Binomial CDF):** What is the probability of **no more** than some number of successes for a pre-determined number of trials?-**no more, no greater than, less than or equal to, and the special case less than**.

  Calculator Commands

  Press 2nd VARS
  ↓B: binomcdf(
  Press ENTER
  trials: Enter the number of trials. **Press ENTER**
  p: Enter the probability of success. **Press ENTER**
  x value: Enter the **maximum** number of successes. **Press ENTER 3 times**

Example: Johnson's statistics exam has 20 true false questions

- What is the probability that he gets no more than 14 correct?

- What is the probability that he gets exactly 14 correct?

- What is the probability that he gets less than 10 correct?

- What is the standard deviation and mean for the number of correct answers?

Notes: Binomial Distribution Scenarios

More Complex Cases for the Binomial CDF

- **(Binomial CDF):** What is the probability of **at least or more than** some number of successes for a pre-determined number of trials? **1-Binomial CDF**
- **(Binomial CDF):** What is the probability that the number of successes for a pre-determined number of trials are between 2 values? **(Binomial CDF of larger value) - (Binomial CDF smaller value)**

Example: Johnson's statistics exam has 20 true false questions

- What is the probability that Johnson gets at least 14 correct?

- What is the probability that Johnson gets at least 10 correct?

- What is the probability that Johnson gets more than 10 correct?

- What is the probability that Johnson gets 9 to 13 questions correct?

- What is the probability that Johnson gets between 9 and 13 questions correct?

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Notes: Normal Approximation of the Binomial

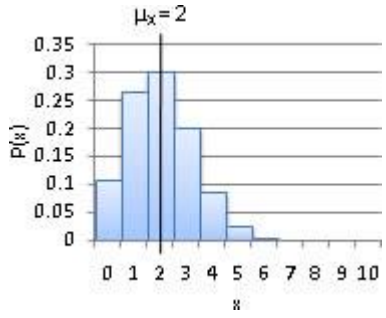
If we have a very large number, the binomial becomes difficult to work with and can exceed the limits of our calculator. When this is the case we can use the normal distribution to approximate the binomial distribution.

Requirements:

1. Two possible outcomes (success and failure).
2. $np \geq 10$ number of successes is greater than 10
3. $nq \geq 10$ number of failures is greater than 10

The best way to understand the effect of n and p on the shape of a binomial probability distribution is to look at some histograms, so let's look at some possibilities.

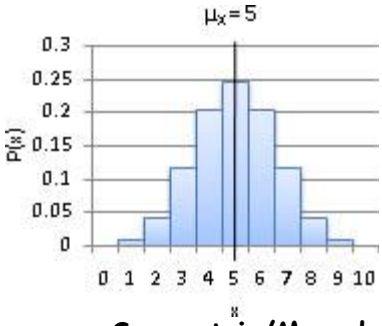
n=10, p=0.2



Skewed Right

Why?

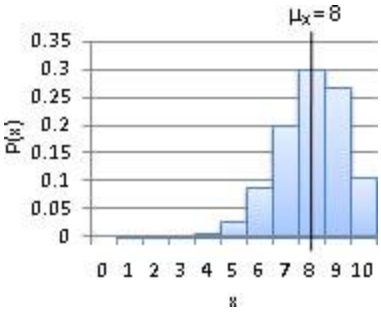
n=10, p=0.5



Symmetric/Mound

Why?

n=10, p=0.8

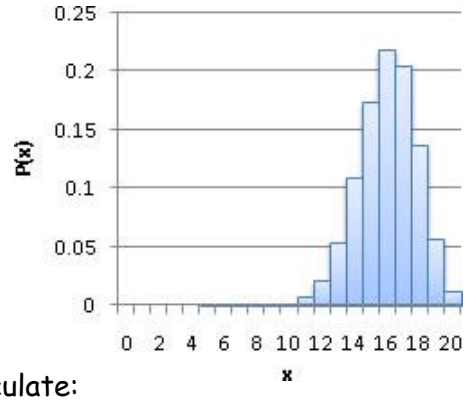


Skewed Left

Why?

Based on these, it would appear that the distribution is symmetric only if p=0.5, but this isn't actually true. Watch what happens as the number of trials, n, increases:

n=20, p=0.8



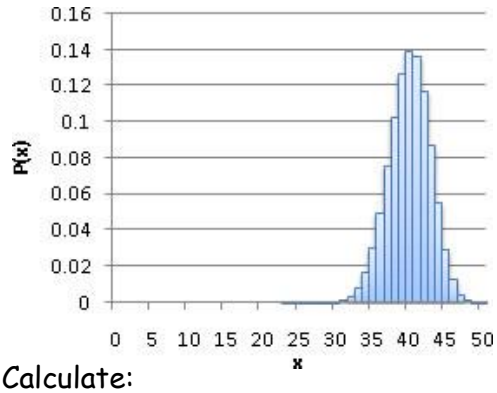
Calculate:

np=

nq=

Is the normal approximation appropriate?

n=50, p=0.8



Calculate:

np=

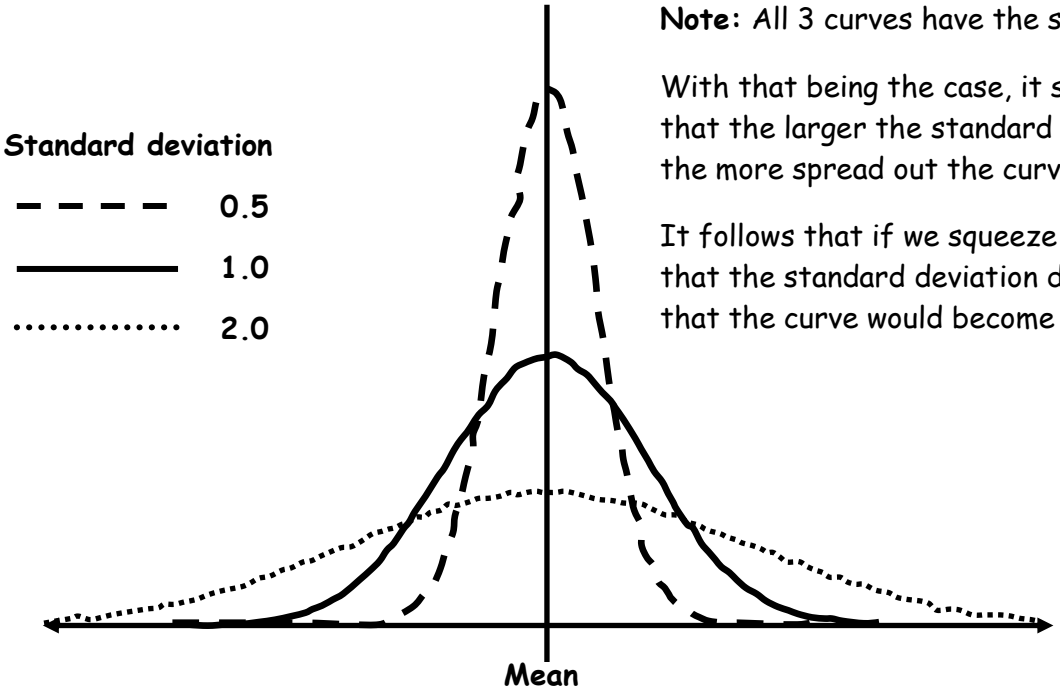
nq=

Is the normal approximation appropriate?

Notes: Introduction to the Normal Curve

A **normal distribution** is an important theoretical idea used in statistics to model data applicable to large populations. The graph of a normal distribution is a symmetrical bell-shaped curve based on the mean and standard deviation of a sample, where the mean, median, and mode are the same. Below are the graphs of normal distributions with the same mean, but different standard deviations.

The mean is located at the center of the graph. Since this is also the median, half of the values lie above the mean and the other half lie below the mean. The shape of the curve is determined by the standard deviation, or spread of the data.



Note: All 3 curves have the same area.

With that being the case, it seems logical that the larger the standard deviation the more spread out the curve is.

It follows that if we squeeze the data so that the standard deviation decreases that the curve would become taller.

The 3 normal curves above show share the same mean but have different standard deviations. This indicative of the fact that the normal curve is actually a family of distributions that follows the equation below.

$$f(x)= \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

There are an infinite number of normal curves because we have an infinite number of means and standard deviations.

The **normal distribution** can describe many things in the world such as scores on tests like the SAT. Characteristics in nature also fall along the normal curve, such as height or weight for adult men and women.

When the mean and standard deviation of a normal distribution are known, then several other values of the distribution can be calculated. It is important to be able to draw a curve and label the mean, as well as three standard deviations above and below the mean. Theoretically, almost all of the data should fall within 3 standard deviations (on each side) of the mean.

Notes: Standardizing with Z-scores

At this point you might be thinking, "Great an infinite number of curves. How am I ever going to learn all of that? I should have been a student aid. Can I go see the counselor?"

Before you commit such an atrocity, you must consider that the normal curve is actually a family of curves and thus they share similar characteristics. Beyond the fact that they all follow the same general equation, they have the same distribution of data in that is distributed as follows:

- 68% of the data falls between plus or minus 1 standard deviations of the mean.
- 95% of the data falls between plus or minus 2 standard deviations of the mean.
- 99.7% of the data falls between plus or minus 3 standard deviations of the mean.

While that helps a great deal, there is still the issue of the fact that the normal curves have different standard deviations and means. For instance, how can I compare Terrence's ACT score to Sofia's SAT score? Answer we use Z-scores.

Recall that we briefly discussed that we can transform very different looking data using z-scores. So let's delve in a little deeper.

Z-scores: A z-score is a ratio that can be used to determine how many standard deviations a value lies from its mean by taking into account a measure of spread (the standard deviation of the distribution) and a measure of center (the mean of the distribution)

For individual values we use: $Z = \frac{x-\mu}{\sigma}$

For sample means we use: $Z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$ or $Z = \frac{\bar{x}-\mu}{\sigma_{\bar{x}}}$ where $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

Z-scores can be computed for any shaped distribution. Consequently, we can use z-scores as a ruler to compare scores from different distributions. We can say that this persons scored 1.2 standard deviations above the mean and this person scored 1.8 standard deviations above the mean and we can do that regardless of the underlying distribution and regardless of the sample size.

Scenario: At a college the scores on the chemistry final exam are approximately normally distributed, with a mean of 75 and a standard deviation of 12. The scores on the calculus final are also approximately normally distributed, with a mean of 80 and a standard deviation of 8. A student scored 81 on the chemistry final and 84 on the calculus final. Relative to the students in each respective class, in which subject did this student do better?

What would a 60 on the Chemistry final equate to on the calculus final?

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Notes: Introduction to the Standard Normal

With the normal distributions we don't have to just compare standard deviations, we can compare percentages, which is a good thing. I don't recall any student of mine ever telling someone that they were 1.2 standard deviations better. We always seem to compare percents. You will recall that we said that all normal curves have their data distributed as follows:

- 68% of the data falls between plus or minus 1 standard deviations of the mean.
- 95% of the data falls between plus or minus 2 standard deviations of the mean.
- 99.7% of the data falls between plus or minus 3 standard deviations of the mean.

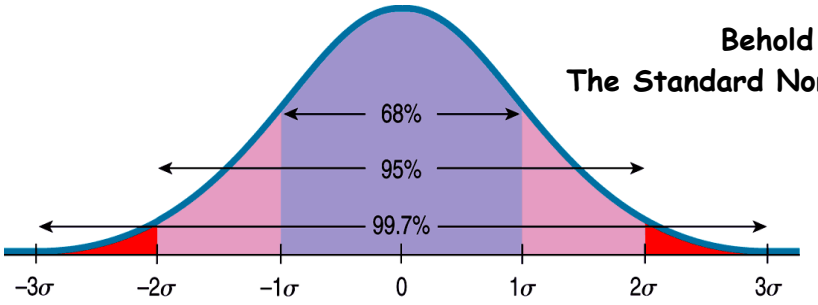
We call this the **Empirical Rule:** you need to remember this.

Empirical Rule: The Empirical Rule provides us with the amount of area under the normal curve or the percentage of data that lies between **± 1**, **± 2**, and **± 3**, standard deviations. These percentages are true for all normal curves regardless of the mean or standard deviation of the normal curve.

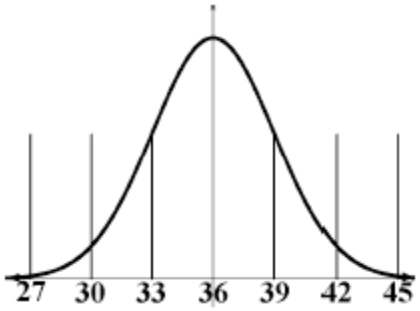
Caution: The empirical rule only applies to normal curves. **Do not** use the empirical rule for any distribution that is not normal.

Because the family of normal curves all follow that distribution of data and because we have the z-score as a standardizing tool, we often use the standard normal which is a normal curve that has been scaled using z-scores and is centered at zero with a standard deviation of 1.

Behold
The Standard Normal Curve



Scenario: Suppose the ages of adults taking a business course for the last 5 years at a local college are normally distributed as shown in the graph at right. Based on the 68-95-99.7 rule we know:



68% of all adults taking the business class are between the ages of _____ and _____.

95% of all adults taking the business class are between the ages of _____ and _____.

99.7% of all adults taking the business class are between the ages of _____ and _____.

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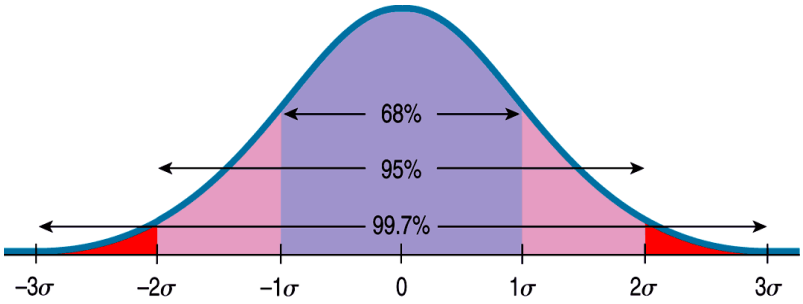
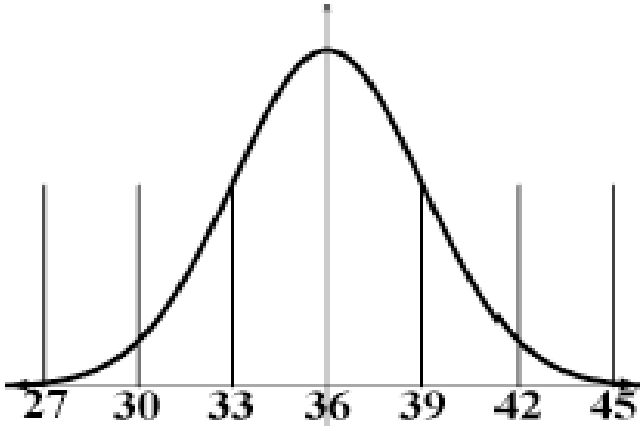
Notes: Introduction to the Standard Normal



Caution: The percentages provided by the Empirical Rule are not percentile ranks. A Percentile rank provides the area from negative infinity up to a specific value.

Percentile Ranks: A percentile rank provides us the amount of area or percent of data that is below a certain value. For a normal curve 68% of the data lies between **±1** standard deviations. However, a z-score of 1 has a percentile rank of 84.13%. This means that 84.13% of the data fall below 1 standard deviations. The difference is that percentile ranks always measure the area from negative infinity up until the value.

Scenario: Suppose the ages of adults taking a business course for the last 5 years at a local college are normally distributed as shown in the graph at right. Based on the 68-95-99.7, calculate the following percentile ranks



27_____

30_____

33_____

36_____

39_____

42_____

45_____

Calculate the following. These are not percentile ranks.

30 to 36_____

36 to 42_____

39 to 45_____

27 to 33_____

27 to 30_____

42 to 45_____

Notes: Normal Distribution Probability (Z-scores)

You should recall that a **z-score** is a ratio that can be used to determine how many standard deviations a value lies from its mean by taking into account a measure of spread (the standard deviation of the distribution) and a measure of center (the mean of the distribution)

For individual values we use: $Z = \frac{x-\mu}{\sigma}$

For sample means we use: $Z = \frac{\overline{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$ or $Z = \frac{\overline{x}-\mu}{\sigma_{\overline{x}}}$ where $\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}}$

Previously we discussed how Z-scores can be computed for any shaped distribution. Consequently, we can use z-scores as a ruler to compare scores from different distributions. We can say that this person **scored 1.2 standard deviations** above the mean and this person **scored 1.8 standard deviations** above the mean and we can do that regardless of the underlying distribution and regardless of the sample size.

We also discussed how we could compute exact percentages when we applied the Empirical rule to the normal distribution. However, not all z-scores are **± 1**, **± 2**, and **± 3**. How can we find the percentile rank for associated with any Z-score. The answer is Calculus. We compute the area under the curve. Fortunately, that work is done for us either through the use of a table or the uses of our calculator.

As it turns out the students who scored 1.2 standard deviations and 1.8 standard deviations above the mean both came from normal populations. **The fact that they came from a normal population is critical**. If they had not come from normal populations, we would just have to shrug our shoulders because we don't have enough information.

Consider the students who scored 1.2 and 1.8 Standard deviations above the mean.

This is what we know:

- The population is normal - given
- The mean is 0 – we are working with a z-score from a normal population (*standard normal distribution*)
- The standard deviation is 1 (*standard normal distribution*)

Steps for σ's 1.2 above

1. **Draw the graph**
   - Label the Mean
   - Label the Upper& Lower
   - Shade
2. **2nd VARS ↓ 2:normalcdf (**
   - Lower: - infinity (-9999999)
   - Upper: 1.2
   - **μ** = 0
   - **σ** = 1

Steps for σ's 1.2 above

1. **Draw the graph**
   - Label the Mean
   - Label the Upper & Lower
   - Shade
2. **2nd VARS ↓ 2:normalcdf (**
   - Lower: -
   - Upper:
   - **μ** =
   - **σ** =

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Notes: Normal Distribution Probability (Z-scores)-Scenarios

1. **Write the Equation**

   $z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$ **or** $z = \frac{x-\mu}{\sigma}$

   z = the number of standard deviations a value is from the mean
   μ = the mean of the population or what is assumed to be true
   $\bar{x}$ = the mean of the sample. **Note: x** = single value or score
   σ = the standard deviation of the population
   n = the size of the sample

2. **Draw the graph**
   - Label the Mean
   - Label the Upper and Lower
   - Shade
3. **Write the probability statement**
4. **2nd VARS ↓ 2:normalcdf (**
   - Lower
   - Upper
   - μ =
   - σ =

Compute the z-scores and calculate the probabilities for the following: The life expectancy of a particular brand of lightbulb is normally distributed with a mean of 1500 hours and a standard deviation of 75 hours

a. What is the probability that the light bulb will last less than 1,410 hours?

b. What is the probability that the light bulb will last more than 1,583 hours?

c. What is the probability that the light bulb will last between 1,563 and 1,648 hours?

d. What is the probability that the light bulb will last between 1,416 and 1,677 hours?

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Notes: Normal Distribution Probability (Actual Values)-Scenarios

Thus far we have been calculating probabilities for the normal using z-scores, however, the calculator permits us to go directly from the actual values to the answer all in one step. The only difference is that we now need to enter the actual mean, standard deviation and value(s) of interest.

1. **Write the Equation**

   $z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$ **or** $z = \frac{x-\mu}{\sigma}$

   z = the number of standard deviations a value is from the mean
   μ = the mean of the population or what is assumed to be true
   $\bar{x}$ = the mean of the sample. **Note: x** = single value or score
   σ = the standard deviation of the population
   n = the size of the sample

2. **Draw the graph**
   - Label the Mean
   - Label the Upper and Lower
   - Shade
3. **Write the probability statement**
4. **2nd VARS ↓ 2:normalcdf (**
   - Lower
   - Upper
   - μ =
   - σ =

Using the actual values calculate the probabilities for the following: A particular brand of lightbulb is **normally distributed** with a mean of 1500 hours and a standard deviation of 75 hours.

a. What is the probability that the light bulb will last less than 1,410 hours?

b. What is the probability that the light bulb will last more than 1,583 hours?

c. What is the probability that the light bulb will last between 1,563 and 1,648 hours?

d. What is the probability that the light bulb will last between 1,416 and 1,677 hours?

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Notes: Inverse Norm (Z-scores)

Thus far, we have been trying find the probability or a percentile rank given a z-score or actual value. We also have the ability to calculate a z-score or an actual value given a percentile rank **if we know that the underlying distribution is normal.**

1. **Write the Equation**

   $\mathbf{z} = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$ **or** $\mathbf{z} = \frac{x-\mu}{\sigma}$

   z = the number of standard deviations a value is from the mean
   μ = the mean of the population or what is assumed to be true
   $\bar{x}$ = the mean of the sample. **Note: x** = single value or score
   σ = the standard deviation of the population
   n = the size of the sample

2. **Draw the graph**
   - Label the percent-be careful
   - Label the Mean
   - Label the Upper and Lower
   - Shade
3. **Write the probability statement**
4. **2nd VARS ↓ 2:normalcdf (**
   - Lower
   - Upper
   - μ =
   - σ =

The Underlying population is **normal** for the examples that follow: **Flip back to page 98**

Example: A student claimed to have scored at the 99th percentile. What is the associated z-score?

Example: A student claimed to have scored at the 25th percentile. What is the associated z-score?

Example: The upper 10% of students are associated with what z-score?

Example: The lowest 10% of students are associated with what z-score?

Example: 80% of students scored above what z-score?

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Notes: Inverse Norm (Actual Values)

Thus far, we have been trying find the probability or a percentile rank given a z-score or actual value. We also have the ability to calculate a z-score or an actual value given a percentile rank **if we know that the underlying distribution is normal.**

1. **Write the Equation**

   $z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$ **or** $z = \frac{x-\mu}{\sigma}$

   z = the number of standard deviations a value is from the mean
   μ = the mean of the population or what is assumed to be true
   $\bar{x}$ = the mean of the sample. **Note: x** = single value or score
   σ = the standard deviation of the population
   n = the size of the sample

2. **Draw the graph**
   - Label the percent-be careful
   - Label the Mean
   - Label the Upper and Lower
   - Shade
3. **Write the probability statement**
4. **2nd VARS ↓ 2:normalcdf (**
   - Lower
   - Upper
   - μ =
   - σ =

For the following questions the population is normal with a mean of 79 and a standard deviation of 7.

Example: A student claimed to have scored at the 99th percentile. What is the score? .

Example: A student claimed to have scored at the 25th percentile. What is the score? .

Ex The upper 10% of students are associated with scores above?

Example: The lowest 10% of students are associated with scores below what? .

Example: 80% of students scored above what?

Notes: Inverse Norm (Actual Values)

Scenario: A packing machine is set to fill a cardboard box with a mean of 16.2 ounces of cereal. Suppose the amounts per box form a normal distribution with a standard deviation equal to 0.1 ounces.

a. What percentage of the boxes will end up with at least 1 pound of cereal?

b. Ten percent of the boxes will contain less than what number of ounces?

c. Eighty percent of the boxes will contain more than what number of ounces?

c. 90% of the boxes will be below what weight?

e. 15% of the boxes are above what weight?

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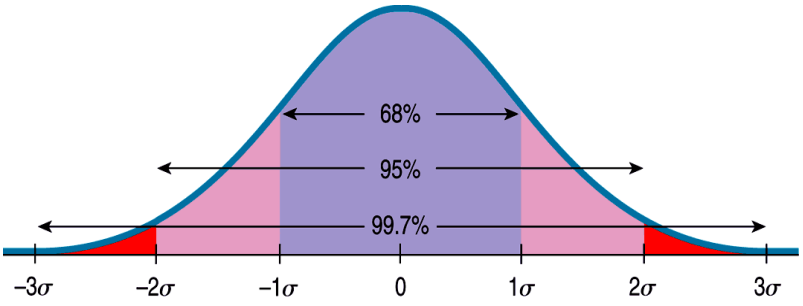
Notes: Data Intervals for the Normal Distribution

We have learned to calculate probabilities based on any z-score that is associated with a normal distribution and we have expanded that ability to calculate the probabilities of actual values. We have also learned used the inverse norm function to work backwards from percentile ranking associated with a normal distribution to find the corresponding z-score or actual value.

Using the skills above and what we know about the Empirical rule we should be able to establish intervals centered about the mean that contain a specific percentage of the data.

Recall the Empirical rule states that normal curves have the following distribution:

- 68% of the data falls between plus or minus 1 standard deviations of the mean.
- 95% of the data falls between plus or minus 2 standard deviations of the mean.
- 99.7% of the data falls between plus or minus 3 standard deviations of the mean.



If via Algebra I rearrange $\mathbf{z = \frac{x-\mu}{\sigma}}$,

I arrive at the following:

$\mathbf{x = \mu - z\sigma}$ and $\mathbf{x = \mu + z\sigma}$

which is most commonly expressed as:

$$\mathbf{x = \mu \pm z\sigma}$$

Thus if I go back to my cereal box that has a mean of 16.2 ounces and a standard deviation of 1 ounce, I should be able to establish where certain weights of the boxes fall.

Use the equations above, to establish lower and upper bounds of an interval containing the following percentages:

Lower Area: $\frac{(1-\%)}{2}$

Upper Area: $\frac{(1-\%)}{2} + \%$

- 68%
- 95%
- 80%

Hint: Draw and label the graph.
Calculate the associated z-scores.

- 98%
- 99%

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Notes: Normal Approximation of the Binomial

The normal distribution is continuous as opposed to the binomial which is discrete. In actuality, the number of people and animals are discrete (they really could be counted) However, the populations are so large that the binomial becomes difficult to work with and can exceed the limits of our calculator, as a consequence we use the normal distribution as an approximation of the binomial. So how large must a population be in order to justify using the normal approximation? To be honest the answer varies, but we will use:

- $np \geq 10$ number of successes is greater than 10
- $nq \geq 10$ number of failures is greater than 10

i. An archer with an 80% bull's eye rate will be shooting 200 arrows.
   a) What are the mean and Standard deviation of the number of bull's eyes she might get?

   b) Is a Normal Model appropriate here? Justify numerically

   c) Use the Empirical rule to describe the distribution as to the number of bull's eyes she might get and draw the picture.

   d) Would you be surprised if she made 140 bull's eyes or less?
   Explain using both a binomial and the normal approximation.

ii. The archer above purchases a new bow and hits 6 consecutive bull's eyes.
   a) Is a normal approximation appropriate for the situation? Justify

   b) Is this compelling evidence that the new bow has changed her bull's eye rate?

Notes: Normal Approximation of the Binomial

The same archer with an 80% bull's eye rate is still shooting arrows.

iii. With her new bow the archer hit 55-bull's eyes in 60 shots.

a) How many shots would you expect her to take to hit her first bulls-eye?

b) Calculate the mean and standard deviation for the above scenario.

c) Is a normal approximation appropriate for the situation? Justify

d) Is this compelling evidence that the new bow has changed her bull's eye rate? **Use the Normal**

iv. Current data indicates that 16% of teens who try marijuana will become addicted. 800 adults who used marijuana as teens were sampled. 165 of these adults were found to be addicted to marijuana.

(a) Explain why a normal approximation is an appropriate for the situation?

(b) Calculate the mean and standard deviation of the distribution.

(c) How many would you expect to have to survey to find the first addict?

(d) Is this compelling evidence that the rate of addiction is different than previously believed? Use both the binomial and the normal approximation.

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Notes: Distribution of the Means / Central Limit Theorem

We have been focusing on the likelihood of getting a single value (n=1) and for the Normal distribution we have been able to create the interval on which we expect the data to lie. Early on in this course we discussed that increasing sample size decreased our sampling error that is to say that the variation was lessened when we increased our sample size. To that end, we are going to begin looking at samples with n greater than 1.

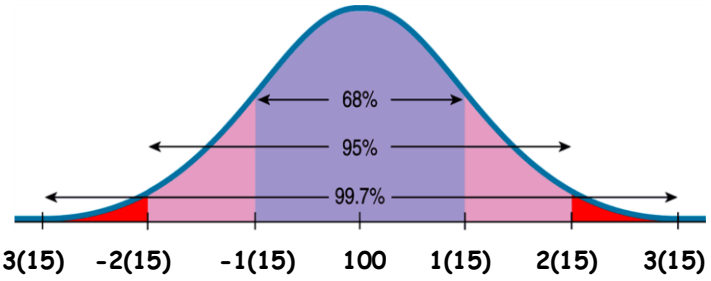
We will begin by considering data that follow a Normal Distribution: When we have a sample size greater than one our formula changes from $\mathbf{z} = \frac{x-\mu}{\sigma}$ to $\mathbf{z} = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$.

You will notice that the x changes to an $\bar{x}$ because we have changed from working with a single value to working with an average of values. As we mentioned earlier, the sampling error diminishes as well and we are no longer working with just σ the standard deviation of the population.

When we are working with the distribution of sample means the standard deviation of the means is: $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$ which reads the standard deviation of the Means equals the standard deviation of the population divided by the square root of the sample size.

So what does this mean and how do I apply it?

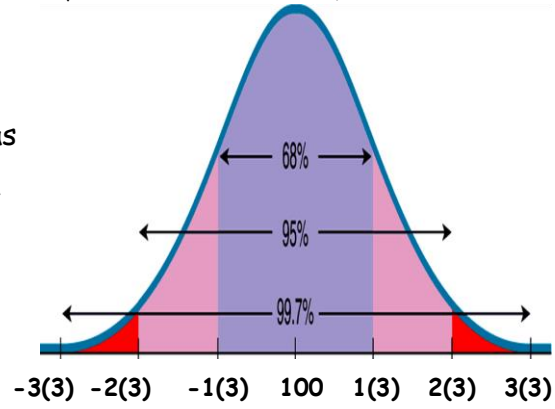
Scenario: Consider that the Distribution of IQ's is normal with a mean of 100 and a standard deviation of 15. The graph of the Empirical Rule is:



Scenario: Now consider the above situation, but with a sample of 25 as opposed to a single value We would still expect the sample mean to be 100.

The **standard deviation** of the sample mean would be: $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$ we would find $\frac{15}{\sqrt{25}} = \frac{15}{5} = \mathbf{3}$

While not to scale, the Graph of the Empirical Rule is narrower in width because of the smaller standard deviation. (Technically, the graph should be one fifth as wide $\frac{1}{\sqrt{25}}$). Because the width is narrower and the area under the curve is still 1, the graph must be taller.



Note: The underlying distribution **Does Not Change**. The underlying distribution has a mean of 100 and a standard deviation of 15.

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Notes: Distribution of the Means / Central Limit Theorem

When we are dealing with an underlying distribution that is normal, the distribution of the sample means is always normal with $\mu = \overline{x}$ and $\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}}$ **regardless of sample size**

Unfortunately, not all distributions are normal. What if the distribution is highly skewed to right as are incomes? Fortunately, we have a solution the **Central Limit Theorem**.

Central Limit Theorem- States that the sampling distribution of the expected values (means) of a population with a mean of μ and a standard deviation of σ will be approximately normal for any population regardless of the shape of the underlying population if the **sample is large enough-(30 or larger)**

The distribution of the sample means is normal with $\mu = \overline{x}$ and $\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}}$ **if n is 30 or larger**

Note: please be aware that the underlying distribution is NOT changing. It is the distribution of the sample means that becomes normal.

Critical Distinction:** Do not confuse the Law of Large numbers with the Central Limit Theorem

- **Law of Large numbers**: as the number of trials increases the percentage of successes moves closer to the expected number of successes-the theoretical number of successes.
- **Central Limit Theorem:** For a large sample the distribution of the means is normal with the following statistics $\mu = \overline{x}$ and $\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}}$

Sampling Distribution of a Sample Mean The sampling distribution of the sample means is approximately normal with mean (μ) and standard deviation $\frac{\sigma}{\sqrt{n}}$

Conditions Necessary to use the Normal for Sample Means

1. Must have a simple random sample
2. The population must be normal **OR** n must be at least 30
3. σ (the standard deviation) must be known for the population
   Given σ (the standard deviation of the population) is known, the standard deviation of the sample means $\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}}$
4. μ. (the population mean) is given

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Notes: Distribution of the Sample Means Scenario

Scenario: The distribution of household incomes in the United States is skewed to the right. The population mean is 71,500 and the standard deviation of the population is 26,200.

a) A sample of 100 is taken. What is shape of the distribution of the sample values along with the mean and standard deviation of the distribution of the sample values?

b) A sample of 25 is taken. What is the shape of the distribution of sample means along with the mean and standard deviation of the distribution of the sample means?

c) A sample of 110 is taken. What is the shape of the distribution of the sample means along with the mean and standard deviation of the distribution of the sample means?

d) A sample of 50 is taken. What is the shape of the distribution of the $\bar{x}$'s along with the mean and standard deviation of the distribution of the $\bar{x}$'s ?

e) What is the probability that a household is randomly selected from the population and earns more than 95,000?

f) What percent of households earn between 55,000 and 85,000?

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Notes: Distribution of the Sample Means Scenarios

1. **Write the Equation**

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

z = the number of standard deviations a value is from the mean
μ = the mean of the population or what is assumed to be true
$\bar{x}$ = the mean of the sample. **Note: x** = single value or score
σ = the standard deviation of the population
n = the size of the sample

2. **Draw the graph**
   - Label the percent-be careful
   - Label the Mean
   - Label the Upper and Lower
   - Shade
3. **Write the probability statement**
4. **2nd VARS ↓ 2:normalcdf (**
   - Lower
   - Upper
   - μ =
   - σ =

Scenario: Suppose that the outstanding credit card balances for young couples are normally distributed with a mean of $650 with a standard deviation of $120.

a. What is the probability that an individual has a credit card balance in excess of 700 dollars?

b. In an SRS of **20** couples, what is the probability that the mean outstanding credit card balance exceeds $700?

c. In an SRS of **80** couples, what is the probability that the mean outstanding credit card balance exceeds $700?

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Notes: Distribution of the Sample Means Scenarios

Scenario: The Wechsler Adult Intelligence Scale (WAIS) is a common "IQ test" for adults. The distribution of WAIS scores for persons over 16 years of age is approximately normal with mean 100 and standard deviation 15.

1. What is the probability that a randomly chosen individual has a WAIS score of 123 or higher?

2. What is the shape of the distribution of sample means along with the mean and standard deviation of the distribution for a SRS of 6 people?

3. What is the probability that a SRS of 6 people has an average score of 123 or higher?

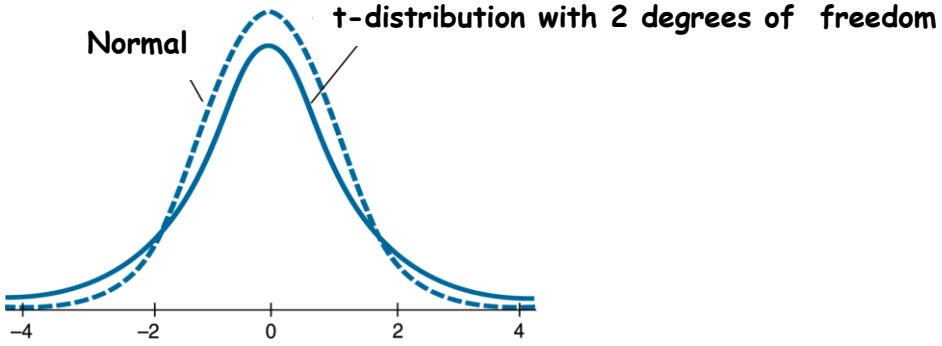
4. A SRS of 40 people was taken. What is the shape of the distribution of the 40 test scores along with the mean and standard deviation?

5. What is the probability that a SRS of 9 people have an average score between 105 and 125?

Notes: T-Distribution & Distribution of Means

One of the requirements for using the normal distribution is that we know the standard deviation of the population which we rarely do. Enter the t distribution (aka, Student t distribution) is the sampling distribution of the *t* statistic.

The distribution of the *t* statistic is similar to the distribution of a standard score/Z-statistic. Both distributions are symmetrical with a mean of zero. Also, both distributions are bell-shaped, although the t-distribution has a larger variance, has more area in the tails, and has less data near the mean.



There are actually many different t-distributions. The particular model or form of the t-distribution is determined by its number of degrees of freedom which is one less than the sample size.

T distribution formula with n-1 degrees of freedom: $t_{df} = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$ or $t_{df} = \frac{\bar{x}-\mu}{S_{\bar{x}}}$ where $S_{\bar{x}} = \frac{s}{\sqrt{n}}$

$\bar{x}$ is the sample mean, μ is the population mean, *s* is the standard deviation of the sample, and n is the sample size.

Properties of the t-distribution

1. The t-distribution is different for different degree of freedom. **Degrees of Freedom equals n-1**
2. The t-distribution is centered at zero and is symmetric about zero.
3. The area under the curve is one.
4. As the sample size increases, the density curve of t-gets closer to the standard normal density curve.
5. The T-distribution has more area in the tails than does the normal.
6. The t-distribution has a greater variance than does the normal

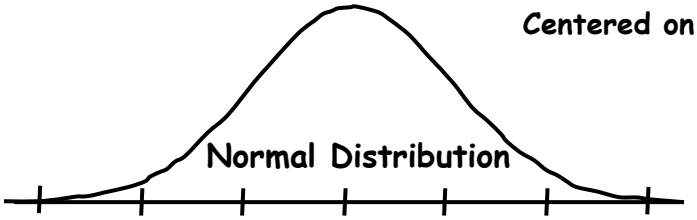
When to use the T-Distribution

1. When the data is **nearly normal (unimodal & symmetric)** and σ the standard deviation of the pop. is not known and there is no clear skewness or outliers it is ok to have a sample size less than 30.
2. When the σ standard deviation of the population is not known and the underlying data does not follow a normal curve the sample size must be 30 or larger.

Note: The t-distribution should *not* be used with small samples n < 30 from populations unless the underlying distribution is approximately normal. **To check your data for normality, you will have to draw boxplots.**

Notes: Comparing the Distribution of Means of the Normal and T-distributions

Centered on a mean ($\bar{x}$) of 0



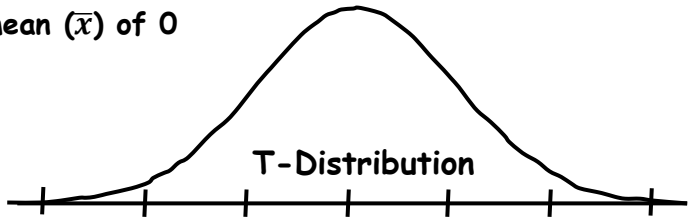
$$\bar{x} \pm z^* \times \frac{\sigma}{\sqrt{n}}$$

Area = 0.68
σ = 5
n = 4

Area = 0.95
σ = 5
n = 4

Area = 0.997
σ = 5
n = 4

Calculator:
2nd VARS
↓3: invNorm(
Press **ENTER**
area: (1-%)÷2
df: n-1



$$\bar{x} \pm t^* \times \frac{s}{\sqrt{n}}$$

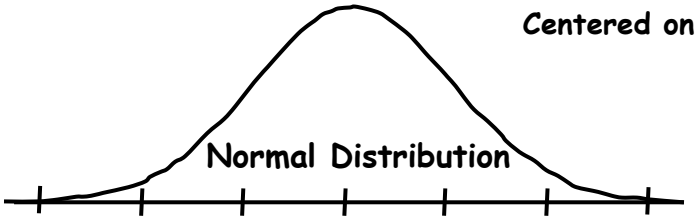
Area = 0.68
s = 5
n = 4

Area = 0.95
s = 5
n = 4

Area = 0.997
s = 5
n = 4

Calculator:
2nd VARS
↓4: invT(
Press **ENTER**
area: (1-%)÷2
df: n-1

Centered on a mean ($\bar{x}$) of 3

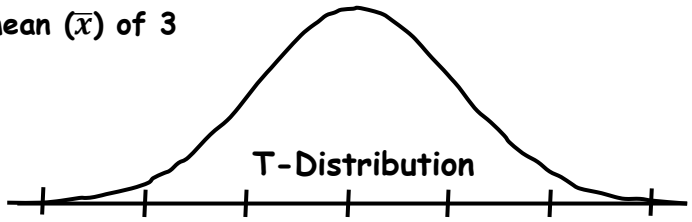


$$\bar{x} \pm z^* \times \frac{\sigma}{\sqrt{n}}$$

Area = 0.68
σ = 5
n = 4

Area = 0.95
σ = 5
n = 4

Area = 0.997
σ = 5
n = 4



$$\bar{x} \pm t^* \times \frac{s}{\sqrt{n}}$$

Area = 0.68
s = 5
n = 4

Area = 0.95
s = 5
n = 4

Area = 0.997
s = 5
n = 4

Notes: Comparing the Distribution of Means of the Normal and T-distributions

Distribution of Sample Means for the T-distribution:

- The set of all sample means is normally distributed
- The mean of the set of sample means **equals** the population mean
- The standard deviation of the sample mean $(\bar{x})$ is **approximately** $s_{\bar{x}} = \frac{s}{\sqrt{n}}$

Scenario 1: A college's data about the incoming freshman indicates that the mean of their high school GPA's was 3.4. The students are randomly assigned to freshman writing seminars in groups of 25. Assume that GPA of the students follow normal distribution.

a. What is the probability that the Mean GPA of the group is less than **3.3** given that the standard deviation of the population is known and is .35?

What is the probability that the Mean GPA of the group is less than 3 given that the standard deviation of the sample is .35?

Calculator:
2nd VARS
↓ 6:tcdf(
Press **ENTER**
Lower:
Upper:
df: n-1

Scenario 2: Assume that the duration of human pregnancies can be described as a normal model with mean 266 days.

a. Suppose a certain obstetrician is currently providing prenatal care to 60 pregnant women. What's the probability that the mean duration of these patients' pregnancies will be less than 260 days if the standard deviation of the population is 16 days?

b. Suppose a certain obstetrician is currently providing prenatal care to 60 pregnant women. What's the probability that the mean duration of these patients' pregnancies will be less than 260 days if the standard deviation of sample is 16 days?

Notes: Confidence Intervals 1-Sample T

The reason we collect data is we usually don't know something about a population and we are forced to take a sample to use as a proxy or estimate of the population parameter of interest.

If we only have sample data, we are never able to say what the exact population mean is. However, we can claim, with a certain level of confidence, that the population mean lies within a specified interval or range. That range is called a confidence interval and is comprised of 2 components the estimate and a margin of error. The estimate or best guess for the population mean is the sample mean. The margin of error creates a range below and above the estimate. The range is based on the sample standard deviation of the sample and the confidence level.

Confidence Intervals: estimate $\pm$ margin of error $\bar{x} \pm t^* \times \frac{s}{\sqrt{n}}$

Estimate: The sample mean ($\bar{x}$) estimates the population parameter **μ**.
Margin of error: For a given confidence level, the maximum distance a sample statistic may lie from the true population parameter

- The **margin of error** is the (t-score)×(the standard error of $\bar{x}$ $SE_{\bar{x}}$) or $t^* \times \frac{s}{\sqrt{n}}$
- S is the standard deviation of the sample and n is the sample size

Because we only have sample data, we are never able to say what the exact population mean is. We can claim, with a certain level of confidence, that the population mean lies within a specified interval.

- **We are ____% confident that the true population mean lies within the interval_____**
- **In repeated testing, we expect this method will capture the true population mean ____% of the time.**

We Cannot Claim:

- ____% of the values lie within the interval—*the interval may be wrong*
- ____% chance that a randomly selected ____will lie within the interval—*the interval may be wrong*
- ____% of samples will result in this interval-*the interval may be wrong*
- ____% probability that the true population mean lies within the interval

(do not use probability; use confidence)

We Can Claim: ____% Confident that the true population mean lies within the interval

We Can Claim: (____% of the intervals collected by this method will capture the true population mean)

Note: We are making a claim about the population mean and not the sample mean.

We do not need to make a claim about the sample mean. The sample mean will be in the center of the interval 100% of the time.

Note:

- As we increase the level of confidence the margin of error or spread increases.

- As we increase alpha the margin of error decreases and our confidence decreases.
- As the sample size increases the margin of error or spread decreases.

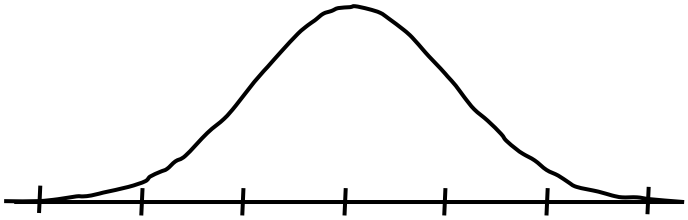
Notes: Confidence Intervals when σ is known

What is a confidence interval anyway?

Consider the empirical rule graph below

(Remember the empirical rule applies the Normal Distribution and not the T-distribution)

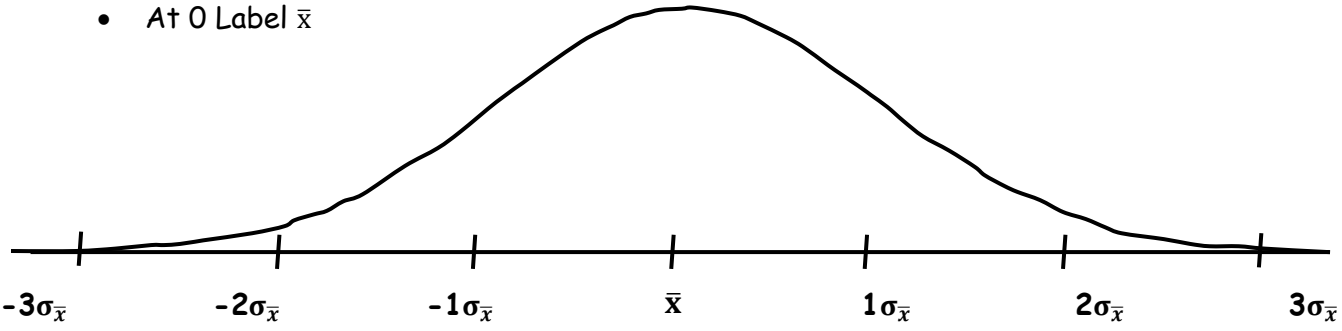
- Label the standard deviations -3σ, -2σ,…3σ
- At 0 Label $\bar{x}$
- If we claim that we are 95% confident, we believe that the true population mean is located somewhere between -2 and +2 standard deviations below or above $\bar{x}$



Making it Real (confidence intervals for a normal distribution)

Consider the empirical rule graph below

- Label the standard deviations $-3\sigma_{\bar{x}}$, $-2\sigma_{\bar{x}}$,…$3\sigma_{\bar{x}}$ **etc.**
- At 0 Label $\bar{x}$



The population standard deviation for the above is known (σ =12.5). The Sample mean ($\bar{x}$) is 77 n=25

- Below $\bar{x}$ record the sample mean
- Calculate $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{12.5}{\sqrt{25}}$
- Below $-3\sigma_{\bar{x}}$ write $-3(\frac{12.5}{\sqrt{25}})$; Below $-2\sigma_{\bar{x}}$ write $-2(\frac{12.5}{\sqrt{25}})$; and continue all the way to $3\sigma_{\bar{x}}$
- Multiply the values and record below each

Create a 68% confidence interval

- Write $\bar{x} \pm 1\sigma_{\bar{x}}$
- Plug in the values
- Solve

Create a 95% confidence interval

- Write $\bar{x} \pm 2\sigma_{\bar{x}}$
- Plug in the values
- Solve

Create a 99.7% confidence interval

- Write $\bar{x} \pm 3\sigma_{\bar{x}}$
- Plug in the values
- Solve

Notes: Confidence Intervals 1-Sample T σ is not known

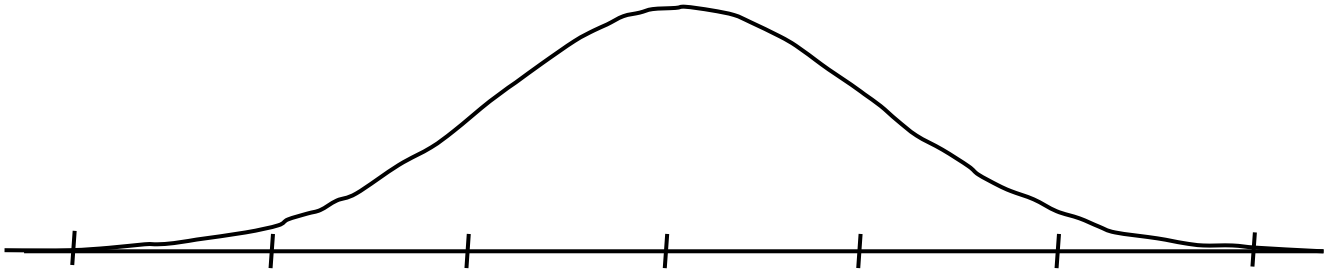
Remember: the empirical rule does not apply for t-distributions because there are many different t-distributions. There is a separate t-distribution for each degree of freedom.

Consider the Following T-distribution

- Sample mean $\bar{x}$ = **77**
- Sample Standard deviation **s** = **12.5**
- Sample size **n** = ***25 (how many degrees of freedom?)***

Making it Real (confidence intervals for a t-distribution)

- In the center place the value of $\bar{x}$
- Below each tick mark record the value of the standard error of $\bar{x}$ or $SE_{\bar{x}} = \frac{s}{\sqrt{n}} =$



You will need to use the following calculator commands to calculate (t*)

- 2nd Vars
- Inverse t
- Area = $\frac{(1-Confidence\ level)}{2}$
- **df** = n-1

Calculate the t-value (t*) associated with a 68% area for a t-distribution n = 25

Calculate the t-value (t*) associated with a 95% area for a t-distribution n = 25

- Calculate the t-value (t*) associated with a 99.7% area for a t-distribution n = 25

Create a 68% confidence interval

- Write $\bar{x} \pm t^* \frac{s}{\sqrt{n}}$
- Plug in the values
- Solve

Create a 95% confidence interval

- Write $\bar{x} \pm t^* \frac{s}{\sqrt{n}}$
- Plug in the values
- Solve

Create a 99.7% confidence interval

- Write $\bar{x} \pm t^* \frac{s}{\sqrt{n}}$
- Plug in the values
- Solve

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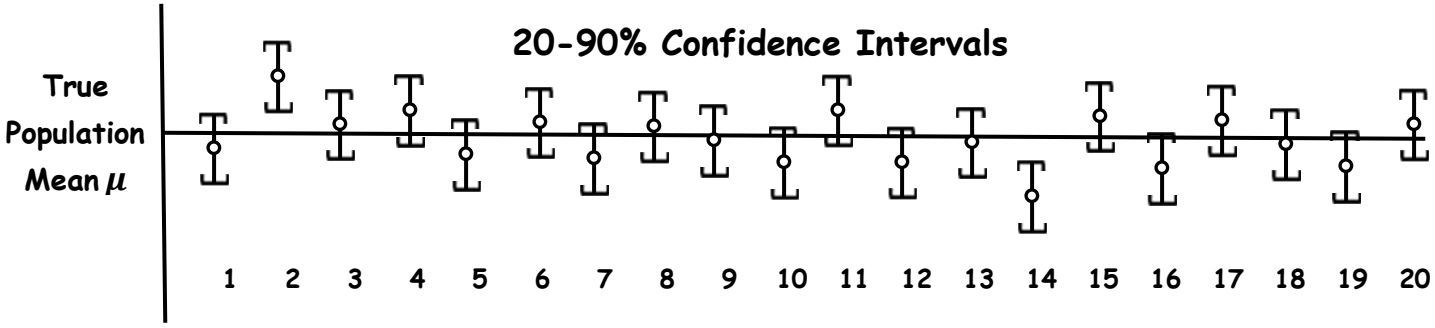
Confidence Intervals for Means σ is known: Z-distribution

The population for the confidence intervals in the chart below is highly skewed. The sample size for each confidence interval is 50, which is greater than 30, therefore the central limit theorem applies. Thus, the distribution of the sample means is approximately normal for each confidence interval.

$\boldsymbol{\sigma}$, the standard deviation of the population is known which means that Z-scores may be used for the critical values. The formula for a confidence interval where $\boldsymbol{\sigma}$ is known is as follows:

$$\bar{x} \pm \mathbf{z}^* \frac{\sigma}{\sqrt{n}} \text{ yields } \textbf{sample mean} \pm \textbf{(1.645)} \frac{\sigma}{\sqrt{50}}$$

Note: Because $\boldsymbol{\sigma}$, the standard deviation of the population, is known, the margin of error for each 90% confidence interval is the same size. While sampling error will not affect the width of the interval when $\boldsymbol{\sigma}$ is known, sampling error does affect the sample mean.



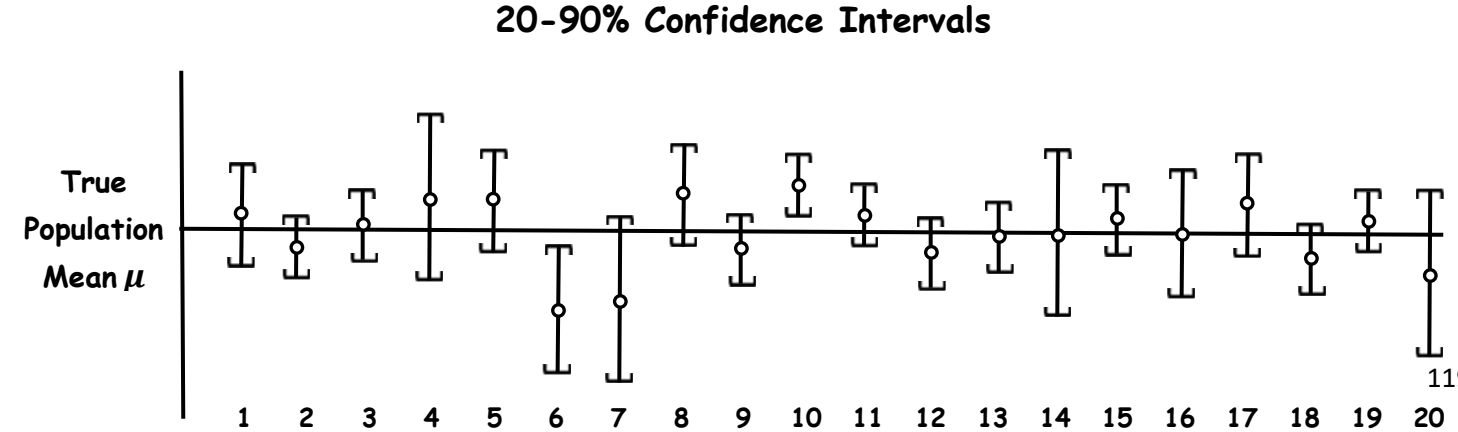
Confidence Intervals for Means σ is *NOT* known s is used: T-distribution

The population for the confidence intervals in the chart below is highly skewed. The sample size for each confidence interval is 50, which is greater than 30, therefore the central limit theorem applies. Thus, the distribution of the sample means is approximately normal for each confidence interval.

$\boldsymbol{\sigma}$, the standard deviation of the population is not known and **s the standard deviation of the sample** is used which means that t-scores will need to be used for the critical values. The formula for a confidence interval is as follows:

$$\bar{x} \pm \mathbf{t}^* \frac{s}{\sqrt{n}} \quad \mathbf{t}^* \text{ with 49 degrees of freedom } \mathbf{= 1.677} \text{ yields } \textbf{sample mean} \pm \textbf{(1.677)} \frac{s}{\sqrt{50}}$$

Note: Because $\boldsymbol{s}$, the standard deviation of the population, is being, the margin of error for each 90% confidence interval varies due to sampling error as does the sample mean.



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Recipe for Success: 1 Sample T-Distribution (Confidence Interval)

1. **Define the parameter μ *in context***
2. **Write the Conditions**
   - **Random Sample**
   - **n < 10% of the population**
   - **Normal Population or *n ≥ 30***

   If data is given, draw a boxplot or histogram-to show normality
3. **Write the Equation**

   $$\overline{x} \pm t^* \left(\frac{s}{\sqrt{n}}\right)$$

   t = the number of standard deviations a value is from the mean
   $\overline{x}$ = the mean of the sample.
   s = the standard deviation of the sample
   n = the size of the sample
4. **Graph and Shade**
5. **Enter the Data if Given**
   - Stat Edit
   - Enter Data in column L1
   - 2nd Quit
   - Stat Calc
   - 1-Var Stats: $\overline{x}$, s_x, **n**
6. **Identify and label all inputs**
   - s comes from the problem or the data
   - $\overline{x}$ comes from the problem or the data
   - **n** comes from the problem or the data
   - **df** = n-1
7. **Calculate *t**
   - 2^{nd} Vars
   - Inverse t
   - Area = $\frac{(1-Confidence\ level)}{2}$
   - **df** = n-1
8. **Plug in the values**
9. **Calculate the Interval**
   - Stat Tests
   - **8:T Interval**
   - Highlight **Stats** *(Highlight Data if data is given)*
   - $\overline{x}$ comes from the problem or the data
   - s_x come from the problem or the data
   - **n** comes from the problem or the data
   - Inter the confidence level
10. **Write the Interval**
11. **Write the Conclusion**

    We are _____% confident that the true population mean for _____ lies within the interval _____. *Restate the definition of the mean*
12. **Explain the meaning of the confidence level-***if asked*

    In repeated sampling, we expect that this method will capture the true population mean _____percent of the time.
    Restate the Confidence Level

Notes: 1 Sample T-Distribution Confidence Interval (Scenario)

Scenario 1: A publishing company has just published a new college textbook. Before the company decides the price at which to sell textbook, it wants to know the average price of such textbook in the market. The research department at the company took a **random sample** of 36 textbook and collected information on their prices. The information produced a mean price of $70.50 for this sample with a sample standard deviation of 4.50. Construct a 90% confidence interval for the price of this type of textbook.

Using the above problem calculate a 95% confidence interval for the type of textbook.

Using the above problem calculate a 99% confidence interval for the type of textbook

What do you notice about the 3 different confidence intervals? Explain the differences.

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Recipe for Success: 1 Sample T-Distribution (Confidence Interval)

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2. **Write the Conditions**
   1. **Random Sample**
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$$\bar{x} \pm t^* \left(\frac{s}{\sqrt{n}}\right)$$

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   - Highlight **Stats** *(Highlight Data if data is given)*
   - $\bar{x}$ comes from the problem or the data
   - s_x come from the problem or the data
   - **n** comes from the problem or the data
   - Inter the confidence level
10. **Write the Interval**
11. **Write the Conclusion**

    We are _____% confident that the true population mean for _____ lies within the interval _____. *Restate the definition of the mean*
12. **Explain the meaning of the confidence level-***if asked*

    In repeated sampling, we expect that this method will capture the true population mean _____percent of the time.
    Restate the Confidence Level

Notes: 1 Sample T-Distribution Confidence Interval (Scenario)

Scenario 2: A professor at a large university believes that students take an average of 15 credit hours per term. A random sample of 24 students in her class of 250 students reported the following number of credit hours that they were taking:

12	**13**	**14**	**14**	**15**	**15**	**15**	**16**	**16**	**16**	**16**	**16**
17	**17**	**17**	**18**	**18**	**18**	**18**	**19**	**19**	**19**	**20**	**21**

Find a 95% confidence interval for the number of credit hours taken by the students in the professor's class. Interpret the interval.

Does this sample indicate that students are taking more credit hours than the professor believes?

Notes: 1 Sample T-Distribution Confidence Interval (Scenario)

Scenario 3: Insurance companies track life expectancy information to assist in determining the cost of life insurance policies. The insurance company knows that, last year, the life expectancy of its policy-holders was 77 years. They want to know if their clients this year have a longer life expectancy, on average, so the company randomly samples some of the recently paid policies to see if the mean life expectancy of policyholders has increased. The insurance company will only change their premium structure if there is evidence that people who buy their policies are living longer than before.

86	**75**	**83**	**84**	**81**	**77**	**78**	**79**	**79**	**81**
76	**85**	**70**	**76**	**79**	**81**	**73**	**74**	**72**	**83**

Does this sample indicate that the insurance company should change its premiums because life expectancy is now different? Calculate a 90% confidence interval and interpret the level.

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Notes: Hypothesis Tests

Hypotheses: an assumption about a population parameter. This assumption may or may not be true

Hypothesis Testing: The formal procedures used to accept or reject a **statistical hypotheses**. A hypothesis test makes a yes/no decision about the plausibility of a parameter value.

- In Statistics, a hypothesis proposes a model for the world. Then we look at the data.
- If the data are consistent with that model, we have no reason to disbelieve the hypothesis.
    - Data consistent with the model *lend support* to the hypothesis, **but do not *prove* it.**
    - If the data is sufficiently inconsistent, **we can reject the model.**

Consider the logic of Jury Trials:

- We assume a defendant is innocent. (this is our hypothesis)
- We retain that hypotheses unless the facts make that claim unlikely beyond a reasonable doubt.
- Then, and only then, we reject the hypothesis of innocence and declare the person guilty.

The same logic used in jury trials is used in statistical tests of hypotheses:

- We begin by assuming that a hypothesis is true.
- Next we consider whether the data are consistent with the hypothesis.
- If data is consistent with the original claim (the null hypothesis), we retain/accept that the null as true. **We never prove the null.** (we didn't gather evidence to prove it; only to disprove it)
- If the data is not consistent with the original claim (the null hypothesis), then like a jury, we determine whether we believe beyond a reasonable doubt that the original hypothesis is false.

P-Value: In statistics, we are able to quantify our level of doubt. The p-value is **a conditional probability** and gives the likelihood that we would get a test statistic as extreme or more extreme given that the original claim/null hypothesis is true.

- **Given that the null is true, there is a ____% chance that we would calculate a test statistic as extreme or more extreme in favor of the alternative hypothesis.**
- When the data are consistent with the model from the null hypothesis, the P-value is high and we are unable to reject the null hypothesis.
    - In that case, we have to "retain" the null hypothesis we started with.
    - We can't claim to have proved it; instead we "*fail to reject the null hypothesis*" when the data are consistent with the null hypothesis model and are within the range of what we would expect from natural sampling variability.
- If the P-value is low enough, we'll "*reject the null hypothesis*," since what we observed would be very unlikely were the null model true.

Testing Hypotheses

- The null hypothesis, which we denote H_0, specifies a population model parameter of interest and proposes a value for that parameter.
    - We might have, for example, H_0: $\mu = 5$
- We want to compare our data to what we would expect given that H_0 is true.
    - We can do this by finding out how many standard deviations away from the claimed null value we are.
- We then ask how likely it is to get results like we did if the null hypothesis were true.

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Notes: Hypothesis Tests

A Trial as a Hypothesis Test

- Hypothesis testing is very much like a court trial.
- The null hypothesis is that the defendant is innocent.
- We then present the evidence—collect data.
- Then we judge the evidence—"Could these data plausibly have happened by chance if the null hypothesis were true?"
    - If they were very unlikely to have occurred, then the evidence raises more than a reasonable doubt in our minds about the null hypothesis.
- Ultimately, we must make a decision. How unlikely is unlikely?
- Some people advocate setting rigid standards—1 time out of 20 (0.05) or 1 time out of 100 (0.01).
- But if *you* have to make the decision, you must judge for yourself in any particular situation whether the probability is small enough to constitute "reasonable doubt."

What to Do with an "Innocent" Defendant

- If the evidence is not strong enough to reject the presumption of innocent, the jury returns with a verdict of "not guilty."
    - The jury does not say that the defendant is innocent.
    - All it says is that there is not enough evidence to convict, to reject innocence.
    - The defendant may, in fact, be innocent, but the jury has no way to be sure
- Said statistically, we will *fail to reject* the null hypothesis.
    - We never declare the null hypothesis to be true, because we simply do not know whether it's true or not.
    - Sometimes in this case we say that the *null hypothesis has been retained*.
- In a trial, the burden of proof is on the prosecution.
- In a hypothesis test, the burden of proof is on the unusual claim.
- The null hypothesis is the ordinary state of affairs, so it's the alternative to the null hypothesis that we consider unusual (and for which we must find sufficient evidence).

There are four basic parts to a hypothesis test:

1. Hypotheses:
    - State the Null: **Parameter =**
    - Alternative hypothesis: **Parameter <** or **Parameter >** or **Parameter ≠** …(2-tailed)
2. Model and Conditions:
    - Give the Name or Formula for the test
    - List the conditions
3. Mechanics:
    - Plug the values into the equation
    - Calculate the test statistic
    - Calculate the p-value
4. Conclusion
    - State whether we reject or fail to reject the null hypothesis
    - The conclusion should be stated in context
    - The conclusion should give the actions that should be taken

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Notes: Hypothesis Test 1-Sample T-test

Vocabulary:

- **Hypothesis:** An assumption about a characteristic about a population. In this case the population parameter that we are testing is (**μ**) the population mean
- **Hypothesis testing:** A method of determining the validity of a hypothesis. Is the claim likely to be true? ***(Note: we never prove that the claim is true or false.)***
- **Null Hypothesis:** This is the claim about the population parameter.
  It is always expressed as H_0: the population parameter = the assumed claim. **It is always** =
  So for means it would appear something like H_0: **μ = .81** (assuming .81 is the claim)

  (Note: the null is based on the concept of innocent until proven guilty-We assume innocence and we assume that the claim is true, & will continue to do so until there is evidence to the contrary)
- **Alternative Hypothesis:** What must concluded if the null hypothesis is found to be unlikely.
  The alternative Hypothesis can be expressed in 3 different manners depending on the question.

  H_A: **μ ≠ the claim**-the population proportion **is different than** the claim

  H_A: **μ > the claim**-the population proportion **is greater than** the claim

  H_A: **μ < the claim**-the population proportion **is less than** the claim

  Regardless of how H_A: is expressed H_0: will always have an = sign

  - **2 tail tests-** a test in which the rejection region is in both tails

    always associated with H_A: μ ≠ the claim

  - **1 tail test-** a test in which the rejection region is in only one of the tails

    Upper Tail Test: H_A: μ > the claim

    Lower Tail Test: H_A: μ < the claim

- **Alpha-(the level of significance)is a predetermined amount of risk or probability** of committing a Type I error that the tester is willing to accept and **is the level of significance** of the test. The alpha level is the amount of area in the rejection region/the area in the tail(s)
- **Critical Value-**the **value that is associated with a given alpha level**. These are the **actual values** that define the rejection regions. They are **predetermined based on a selected alpha.**
- **test statistic- the value that is computed from our sample data.**
- **p-value-**A conditional probability that tells us the likelihood that our data would occur **given that the null hypothesis is true**. In other words given the null really is actually correct, how likely are we to find the **test statistic** from the sample of the population.

  The p-value is what tells us whether to reject or not.

We reject if the p-value is less than alpha. This is an always and forever amen.

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Recipe for Success: 1 Sample T-Distribution (Hypothesis Test)

1. **Write your Hypothesis**
   - Null H_0: $\mu =$
   - Alternative H_A: $\mu \neq$ or $<$ or $>$
2. **Define the parameter μ *in context***
3. **Write the Conditions**
   1. **Random Sample**
   2. **n < 10% of the population**
   3. **Normal Population or *n > 30***

   If data is given, draw a boxplot or histogram-to show normality
4. **Write the Equation**

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

t = the number of standard deviations a value is from the mean
μ = the mean of the population or what is assumed to be true
$\bar{x}$ = the mean of the sample.
s = the standard deviation of the sample
n = the size of the sample

5. **Draw the graph and Shade**
6. **Enter Data** (if given)
   - Stat Edit
   - Enter Data in column L1
   - 2nd Quit
   - Stat Calc
   - 1-Var Stats: $\bar{x}$, s_x, **n**
7. **List & Label all of input values**
8. **Plot values into the equation**
9. **Calculate the t and the p-value**

   df= n-1

   (df is the degrees of freedom)
   - Stat Tests
   - **2:T-Test Enter**
   - Highlight **Stats** *(Highlight Data if data is given)*
   - μ comes from the problem
   - $\bar{x}$ comes from the problem or the data
   - s_x come from the problem or the data
   - **n** comes from the problem or the data
   - Choose $\neq$ or $<$ or $>$ (use Shaded graph of H_A)
10. **State the Decision**
    - The p-value is_____
    - If the p-value is less than alpha, Reject the Null
    - If the p-value is greater than alpha, Fail to reject the Null
11. **Write the Conclusion**

Reject the Null: Our p-value is _____. We reject the Null. There is sufficient evidence at alpha = ___ to suggest that the true population mean for _____is _____

Restate H_A $\neq$ or $<$ or $>$ mean *Restate the definition of the mean*

Fail to Reject the Null: Our p-value is _____. We Fail to reject the Null. There is not sufficient evidence at alpha = ___ to suggest that the true population mean for _____is _____

Restate the definition of the mean *Restate H_A $\neq$ or $<$ or $>$ mean*

Notes: 1 Sample T-Distribution Hypothesis Test (Scenario)

Scenario 1: The EPA reports that the exhaust emissions for a certain car model has a normal distribution with a mean of 1.45 grams of nitrous oxide per mile. The car manufacturer claims their new process reduces the mean level of exhaust emitted for this car model. A SRS of 28 cars is taken. The mean level of exhaust emitted for this sample is 1.27 grams with a standard deviation of 0.4. Is there statistical evidence at an alpha level of .05 to support the car manufacturer's claim?

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Restate H_A $\neq$ or $<$ or $>$ mean *Restate the definition of the mean*

Fail to Reject the Null: Our p-value is _____. We Fail to reject the Null. There is not sufficient evidence at alpha = ___ to suggest that the true population mean for _____is _____

Restate the definition of the mean *Restate H_A $\neq$ or $<$ or $>$ mean*

Notes: 1 Sample T-Distribution Hypothesis Test (Scenario)

Scenario 2: A credit card company wondered whether giving frequent flyer miles for every purchase would increase card usage, which has a current mean of $2500 per year. They gave free miles to a SRS of 37 credit card customers and found the sample mean to be $2542 with a standard deviation of $109. Is there statistical evidence at an α level of .01 to show that card usage has increased?

Notes: 1 Sample T-Distribution Hypothesis Test (Scenario)

Scenario 3: A steel supplier claims that their bars have elongation of 40 percent under a stress of 10 metric tons. A random sample of 12 steel bars was taken from a warehouse and the elongation measured in percent per 2 inches determined.

Observed values of **39 41 35 33 35 39 33 37 36 40 41 42** were obtained.

Is there evidence at a significance level of .05 to demonstrate that the elongation for the steel bars was different from 40?

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Notes: Hypothesis Tests (Review)

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- We assume a defendant is innocent. (this is our hypothesis)
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- We then ask how likely it is to get results like we did if the null hypothesis were true.

Notes: Hypothesis Tests (Review)

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- Hypothesis testing is very much like a court trial.
- The null hypothesis is that the defendant is innocent.
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- Then we judge the evidence—"Could these data plausibly have happened by chance if the null hypothesis were true?"
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- If the evidence is not strong enough to reject the presumption of innocent, the jury returns with a verdict of "not guilty."
    - The jury does not say that the defendant is innocent.
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- In a trial, the burden of proof id on the prosecution.
- In a hypothesis test, the burden of proof is on the unusual claim.
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There are four basic parts to a hypothesis test:

1. Hypotheses:
    - State the Null: **Parameter =**
    - Alternative hypothesis: **Parameter <** or **Parameter >** or **Parameter ≠** …(2-tailed)
2. Model and Conditions:
    - Give the Name or Formula for the test
    - List the conditions
3. Mechanics:
    - Plug the values into the equation
    - Calculate the test statistic
    - Calculate the p-value
4. Conclusion
    - State whether we reject or fail to reject the null hypothesis
    - The conclusion should be stated in context
    - The conclusion should give the actions that should be taken

Notes: Paired T-Distribution

What is Paired Data?
Data are paired when the observations are collected in pairs (2 samples) or the observations in one group are naturally related to observations in the other group.

How does Paired Data Arise? (How do I recognize that the 2 samples are paired?)

- A subject is measured Before and After treatments and the difference is computed for each subject and then the average of the differences is computed
- A subject is tested twice with different treatments and the difference is computed for each subject and then the average of the differences is computed
- Pairing may also be the comparison of naturally occurring **Couples or Pairs**
    - Twins-One twin would receive treatment 1 and the other twin would receive treatment 2 and the difference in responses would be computed for each set of twins.
    - Spouses- The husband is compared to his wife and the differences between the husband's and wife's response are computed.
    - Siblings- A sibling is compared to his/her sibling and the differences between each sibling pair's response is computed.

Note: For a matched pair design we are looking at the difference between each pair. Those differences will be summed and then an average of those differences will be computed to create our sample average ($\bar{x}_d$).

In order to be a matched pair test the list of the two samples must be of equal length—obviously.

We have worked with Paired Data before:
Recall when we created matched pairs designed experiments for waterproof boots and for mosquito repellant. Why did we do that?

What a Paired T-Test Does
A paired t-test looks at the difference between paired values in two samples in cases where each value in one sample has a natural partner in the other. In a paired t-test, we calculate the differences for each pair. In doing so, we have taken two samples and created a single sample of differences. It is that single sample of differences that we use for all of our statistical calculations.

Paired Data is very powerful and should be used anytime that it is available. Why because paired data controls for variation within the pairs. In other words, we recognize that if data has a pair then an association exists and the samples are not independent. However, the distribution of the differences can still be independent.

Caution:
If the data is paired, **we do not take the mean of each sample and then compare the means**. That type of test is a 2-sample test for means which will be discussed later. **We do take the mean of the paired differences**

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Recipe for Success: Hypothesis Test for a Paired T-test

1. **Write your Hypothesis**
   - Null H_0: $\mu_d = 0$
   - Alternative H_A: $\mu_d \neq$ or $<$ or > 0
2. **Define parameter μ_d *in context* & write the conditions**
   1. **Random Sample**
   2. **n < 10% of the population**
   3. **Differences are independent**
   4. **The differences are Normal or *n > 30***

   If data is given, draw a boxplot or histogram-to show normality
3. **Write the Equation**

$$t = \frac{\bar{x}_d - \mu_d}{\frac{s_d}{\sqrt{n}}}$$

t = the number of standard deviations a value is from the mean
$\mu_d = 0$
$\bar{x}_d$ = the mean of the sample differences
s_d = the standard deviation of the sample differences
n = the number of sample differences

4. **Enter Data** (if given)
   - **Stat Edit**
   - **Enter Data** in columns L_1 & L_2
5. **Find $\bar{x}_d$**
   - At the top of L_3 type 2^{nd} L_1 - 2^{nd} L_2
6. **List & Label all of input values**
   n, $\bar{x}_d$, s_d
   $\mu_d = 0$ & **df= n-1**
   - **Stat Calc**
   - **1-Var Stats** press **Enter**
   - **List** type 2^{nd} L_3
7. **Plug values into the equation**
8. **Calculate the t and the p-value**
   $\mu_d = 0$
   df= n-1
   - Stat Tests
   - **2**:T-Test Enter
   - Highlight **Data** if data is used otherwise highlight **STATS**
   - s_d come from the problem or the data
   - $\bar{x}_d$ comes from the problem or the data
   - **n** comes from the problem or the data
   - Choose $\neq$ or $<$ or $>$ (look for key words)
9. **State the Decision**
   - The p-value is____
   - If the p-value is less than alpha, Reject the Null
   - If the p-value is greater than alpha, Fail to reject the Null
10. **Write the Conclusion**

Reject the Null: Our p-value is _____. We reject the Null. There is sufficient evidence at alpha = _____ to suggest that the true population mean differences for _____is _____

Restate the definition of the mean differences *Restate H_A $\neq$ or $<$ or $>$ mean differences*

Fail to Reject the Null: Our p-value is ____. We Fail to reject the Null. There is not sufficient evidence at alpha = ___ to suggest that the true population mean difference for _____is _____

Restate the definition of the mean difference *Restate H_A $\neq$ or $<$ or $>$ mean difference*

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Notes: Hypothesis Test for a Paired T-test (Scenario)

Scenario 1: One indicator of physical fitness is resting pulse rate. Fourteen men volunteered to test an exercise device advertised on television by using it three times a week for 20 minutes. Their resting pulse rate beats per minute (bpm) were measured before the test began, and then again after six weeks. Results are shown in the table below.

Subject		Alec	Tripp	Matt	Peter	Chris	Cory	Sebastian	Luke	Ture	Winston	Kevin	Proloy	Alex	Tim
Beats per min.	**Before**	**73**	**84**	**83**	**85**	**87**	**91**	**87**	**99**	**85**	**82**	**83**	**79**	**80**	**77**
	After	**73**	**82**	**79**	**81**	**86**	**87**	**84**	**91**	**83**	**79**	**84**	**76**	**77**	**77**

Is there evidence that this kind of exercise can reduce resting pulse rates?

Recipe for Success: Confidence Interval for a Paired T-test

1. **Define parameter μ_d *in context***

2. **Write the Conditions**
   1. **Random Sample**
   2. **n < 10% of the population**
   3. **Differences are independent**
   4. **The differences are Normal or *n > 30***

   If data is given, draw a boxplot/ histogram-to show normality

3. **Write the formula for the Test**

   $$\overline{x}_d \pm t^* \frac{s_d}{\sqrt{n}}$$

   t* = the number of standard deviations a value is from the mean & is based on the Confidence Level
   μ_d = 0
   $\overline{x}_d$ = the mean of the sample differences
   s_d = the standard deviation of the sample differences
   n = the number of sample differences

4. **Enter the Data if Given**
   - **Stat Edit**
   - **Enter Data** in columns **L_1 & L_2**

5. **Find $\overline{x}_d$**
   - **Stat Edit**
   - At the top of **L_3** type **2^{nd} L_1 - 2^{nd} L_2**

6. **Identify & label all inputs.**
   - s_d come from the problem or the data
   - $\overline{x}_d$ comes from the problem or the data
   - **n** comes from the problem or the data
   - **df= n-1** *(df is the degrees of freedom)*

7. **Calculate *t****
   - 2^{nd} Vars
   - Inverse t
   - Area = $\frac{(1-Confidence\ level)}{2}$
   - **df** = n-1

8. **Plug in and calculate the Confidence Interval**
   - **STAT**
   - **TESTS**
   - 8: **T Interval**

9. **Write the Interval**

10. **Write the Conclusion**

We are _____% confident that the true population mean difference for _____ lies within the interval _____.

Restate the definition of the mean differences

11. **Explain the meaning of the confidence level-***if asked*

In repeated sampling we expect this method to capture the true population mean difference for _____ _____% of the time

Restate the definition of the mean differences

Notes: Confidence Interval for a Paired T-test (Scenario)

Scenario 1: One indicator of physical fitness is resting pulse rate. Fourteen men volunteered to test an exercise device advertised on television by using it three times a week for 20 minutes. Their resting pulse rate beats per minute (bpm) were measured before the test began, and then again after six weeks. Results are shown in the table below.

Subject		Alec	Tripp	Matt	Peter	Chris	Cory	Sebastian	Luke	Ture	Winston	Kevin	Proloy	Alex	Tim
Beats per min.	**Before**	**73**	**84**	**83**	**85**	**87**	**91**	**87**	**99**	**85**	**82**	**83**	**79**	**80**	**77**
	After	**73**	**82**	**79**	**81**	**86**	**87**	**84**	**91**	**83**	**79**	**84**	**76**	**77**	**77**

Create a 95% confidence interval for the mean difference and explain the meaning of the confidence level?

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Recipe for Success: Hypothesis Test for a Paired T-test

1. **Write your Hypothesis**
   - Null H_0: $\mu_d = 0$
   - Alternative H_A: $\mu_d \neq$ or $<$ or > 0
2. **Define parameter μ_d *in context* & write the conditions**
   1. **Random Sample**
   2. **n < 10% of the population**
   3. **Differences are independent**
   4. **The differences are Normal or *n > 30***

   If data is given, draw a boxplot or histogram-to show normality
3. **Write the Equation**

$$t = \frac{\bar{x}_d - \mu_d}{\frac{s_d}{\sqrt{n}}}$$

t = the number of standard deviations a value is from the mean
μ_d = 0
$\bar{x}_d$ = the mean of the sample differences
s_d = the standard deviation of the sample differences
n = the number of sample differences

4. **Enter Data** (if given)
   - **Stat Edit**
   - **Enter Data** in columns **L_1 & L_2**
5. **Find $\bar{x}_d$**
   - At the top of **L_3** type **2^{nd} L_1 - 2^{nd} L_2**
6. **List & Label all of input values**
   - **Stat Calc**
   - **1-Var Stats** press **Enter**
   - **List** type **2^{nd} L_3**

   n, $\bar{x}_d$, s_d
   μ_d = 0 & **df= n-1**
7. **Plug values into the equation**
8. **Calculate the t and the p-value**
   μ_d = 0
   df= n-1
   - Stat Tests
   - **2:**T-Test Enter
   - Highlight **Data** if data is used otherwise highlight **STATS**
   - **s_d** come from the problem or the data
   - **$\bar{x}_d$** comes from the problem or the data
   - **n** comes from the problem or the data
   - Choose $\neq$ or $<$ or $>$ (look for key words)
9. **State the Decision**
   - The p-value is____
   - If the p-value is less than alpha, Reject the Null
   - If the p-value is greater than alpha, Fail to reject the Null
10. **Write the Conclusion**

Reject the Null: Our p-value is _____. We reject the Null. There is sufficient evidence at alpha = _____ to suggest that the true population mean differences for

_____is _____

Restate the definition of the mean differences *Restate H_A $\neq$ or $<$ or $>$ mean differences*

Fail to Reject the Null: Our p-value is ____. We Fail to reject the Null. There is not sufficient evidence at alpha = ___ to suggest that the true population mean difference for

_____is _____

Restate the definition of the mean difference *Restate H_A $\neq$ or $<$ or $>$ mean difference*

Date:_____ Compiled by: Loren L. Spencer

Notes: Hypothesis Test for a Paired T-test (Scenario)

Scenario 2: Professor Andy Neill measured the time (in seconds) required to catch a falling meter stick for 12 randomly selected students' dominant hand and non-dominant hand. Professor Neill claims that the reaction time in an individual's dominant hand is less than the reaction time in their non-dominant hand.

Student	1	2	3	4	5	6	7	8	9	10	11	12
Dominant Hand	0.177	0.210	0.186	0.189	0.198	0.194	0.160	0.163	0.166	0.152	0.190	0.172
Non-dominant	0.179	.202	.208	.184	.215	.193	.194	.160	.209	.164	.210	.197

Test an appropriate hypothesis and state your conclusion

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Recipe for Success: Confidence Interval for a Paired T-test

1. **Define parameter μ_d *in context***

2. **Write the Conditions**
   1. **Random Sample**
   2. **n < 10% of the population**
   3. **Differences are independent**
   4. **The differences are Normal or *n > 30***

   If data is given, draw a boxplot/ histogram-to show normality

3. **Write the formula for the Test**

   $$\bar{x}_d \pm t^* \frac{s_d}{\sqrt{n}}$$

   t* = the number of standard deviations a value is from the mean & is based on the Confidence Level
   μ_d = 0
   $\bar{x}_d$ = the mean of the sample differences
   s_d = the standard deviation of the sample differences
   n = the number of sample differences

4. **Enter the Data if Given**
   - **Stat Edit**
   - **Enter Data** in columns **L_1 & L_2**

5. **Find $\bar{x}_d$**
   - **Stat Edit**
   - At the top of **L_3** type **2^{nd} L_1 - 2^{nd} L_2**

6. **Identify & label all inputs.**
   - s_d come from the problem or the data
   - $\bar{x}_d$ comes from the problem or the data
   - **n** comes from the problem or the data
   - **df= n-1** *(df is the degrees of freedom)*

7. **Calculate *t****
   - 2^{nd} Vars
   - Inverse t
   - Area = $\frac{(1-Confidence\ level)}{2}$
   - **df** = n-1

8. **Plug in and calculate the Confidence Interval**
   - **STAT**
   - **TESTS**
   - 8: **T Interval**

9. **Write the Interval**

10. **Write the Conclusion**

We are _____% confident that the true population mean difference for _____ lies within the interval _____.

Restate the definition of the mean differences

11. **Explain the meaning of the confidence level-***if asked*

In repeated sampling we expect this method to capture the true population mean difference for _____ _____% of the time

Restate the definition of the mean differences

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Notes: Confidence Interval for a Paired T-test (Scenario)

Scenario 2: Professor Andy Neill measured the time (in seconds) required to catch a falling meter stick for 12 randomly selected students' dominant hand and non-dominant hand. Professor Neill claims that the reaction time in an individual's dominant hand is less than the reaction time in their non-dominant hand.

Student	1	2	3	4	5	6	7	8	9	10	11	12
Dominant Hand	0.177	0.210	0.186	0.189	0.198	0.194	0.160	0.163	0.166	0.152	0.190	0.172
Non-dominant	0.179	.202	.208	.184	.215	.193	.194	.160	.209	.164	.210	.197

Create a 97% confidence interval for the mean difference and explain the confidence level?

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Recipe for Success: Hypothesis Test for a Paired T-test

1. **Write your Hypothesis**
   - Null H_0: $\mu_d = 0$
   - Alternative H_A: $\mu_d \neq$ or $<$ or > 0
2. **Define parameter μ_d *in context* & write the conditions**
   1. **Random Sample**
   2. **n < 10% of the population**
   3. **Differences are independent**
   4. **The differences are Normal or *n > 30***

   If data is given, draw a boxplot or histogram-to show normality
3. **Write the Equation**

$$t = \frac{\bar{x}_d - \mu_d}{\frac{s_d}{\sqrt{n}}}$$

   t = the number of standard deviations a value is from the mean
   $\mu_d = 0$
   $\bar{x}_d$ = the mean of the sample differences
   s_d = the standard deviation of the sample differences
   n = the number of sample differences
4. **Enter Data** (if given)
   - **Stat Edit**
   - **Enter Data** in columns **L_1 & L_2**
5. **Find $\bar{x}_d$**
   - At the top of **L_3** type **2^{nd} L_1 - 2^{nd} L_2**
6. **List & Label all of input values**
   n, $\bar{x}_d$, s_d
   μ_d = 0 & **df= n-1**
   - **Stat Calc**
   - **1-Var Stats** press **Enter**
   - **List** type **2^{nd} L_3**
7. **Plug values into the equation**
8. **Calculate the t and the p-value**
   μ_d = 0
   df= n-1
   - Stat Tests
   - **2:**T-Test Enter
   - Highlight **Data** if data is used otherwise highlight **STATS**
   - **s_d** come from the problem or the data
   - $\bar{x}_d$ comes from the problem or the data
   - **n** comes from the problem or the data
   - Choose $\neq$ or $<$ or $>$ (look for key words)
9. **State the Decision**
   - The p-value is_____
   - If the p-value is less than alpha, Reject the Null
   - If the p-value is greater than alpha, Fail to reject the Null
10. **Write the Conclusion**

Reject the Null: Our p-value is _____. We reject the Null. There is sufficient evidence at alpha = _____ to suggest that the true population mean differences for

_____is _____

Restate the definition of the mean differences *Restate H_A $\neq$ or $<$ or $>$ mean differences*

Fail to Reject the Null: Our p-value is ____. We Fail to reject the Null. There is not sufficient evidence at alpha = ___ to suggest that the true population mean difference for

_____is _____

Restate the definition of the mean difference *Restate H_A $\neq$ or $<$ or $>$ mean difference*

Notes: Hypothesis Test for a Paired T-test (Scenario)

Scenario 3: 2007 Problem 4 Investigators at the U.S. Department of Agriculture wished to compare methods of determining the level of *E. coli* bacteria contamination in beef. Two different methods (A and B) of determining the level of contamination were used to each of ten randomly selected specimens of a certain type of beef. The data obtained, in milli-microbes/liter of ground beef, for each of the methods are shown in the table below.

	Specimen									
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
A	22.7	23.6	24.0	27.1	27.4	27.8	34.4	35.2	40.4	46.8
B	23.0	23.1	23.7	26.5	26.6	27.1	33.2	35.0	40.5	47.8

Is there a significant difference in the mean amount of *E.coli* bacteria detected by the two methods for this type of beef? Provide a statistical justification to support your answer.

Notes: Confidence Interval for a Paired T-test (Scenario)

Scenario 3: 2007 Problem 4 Investigators at the U.S. Department of Agriculture wished to compare methods of determining the level of *E. coli* bacteria contamination in beef. Two different methods (A and B) of determining the level of contamination were used to each of ten randomly selected specimens of a certain type of beef. The data obtained, in milli-microbes/liter of ground beef, for each of the methods are shown in the table below.

	Specimen									
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
A	22.7	23.6	24.0	27.1	27.4	27.8	34.4	35.2	40.4	46.8
B	23.0	23.1	23.7	26.5	26.6	27.1	33.2	35.0	40.5	47.8

Create a 90% Confidence Interval for the mean difference. Does your confidence interval indicate that the detection methods are different? Justify your response

a

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Notes: Type I & Type II Errors

Review of Concepts:

How to Think About P-Values

- A P-value is a conditional probability
- The probability of getting a test statistic as extreme or more extreme given that the null hypothesis is true.
- There is a _____% chance that we would get a test statistic this extreme in favor
  P-value

  of_____ when in fact_____ is true.
  Restate H_A *Restate* H_0

  Note: The P-value is NOT the probability that the null hypothesis is true.

How to Think About Alpha Levels

- Sometimes we need to make a firm decision about whether or not to reject the null hypothesis.
- When the P-value is small, it tells us that our data **are rare** *given the null hypothesis*.
- We can define a "rare event" by arbitrarily setting a threshold for our P-value.
  - If our P-value falls below that point, we will reject the null in favor of the alternative. We call such results statistically significant.
  - The threshold is called an alpha level, denoted by α.
- The alpha level is also called the significance level. When we reject the null hypothesis, we say that the test is "significant at that level."
- What can you say if the P-value is large and does not fall below α?
  - You should say: "The data have failed to provide sufficient evidence to reject the null hypothesis."
  - Do not say that you "accept the null hypothesis."
  - Recall that, in a jury trial, if we do not find the defendant guilty, we say the defendant is "not guilty"—we don't say that the defendant is "innocent."

Sampling Variation and Alpha

- The alpha level measures the risk that we are willing to take in rejecting a true null.
  - An alpha level of .05 means that we expect to incorrectly reject the null hypothesis 5% of the time.
  - This error is due to the natural variations that comes from sampling—sampling variation
  - Obviously, different people and different situations will result in different significance levels or alpha levels
    - What you consider to be statistically significant might not be the same as what someone else considers statistically significant.

  Note: Different alpha levels may be used, but each test will give only one P-value.

Why Wouldn't We Choose a very small alpha to minimize the chance of mistakenly rejecting a true null?

- Answer: Because it becomes very difficult to correctly reject a false null.
- Selecting alpha is a balancing act between 2 types of errors:
  1. **Rejecting a true null and**
  2. **Failing to reject a false null**

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Notes: Type I & Type II Errors

Making Errors

- Here's some shocking news: nobody's perfect. Even with lots of evidence we can still make the wrong decision. This error is due to sampling variation
- When we perform a hypothesis test, we can make mistakes in *two* ways:
  1. The null hypothesis is true, but we mistakenly reject it. (Type I error or alpha-α)
  2. The null hypothesis is false, but we fail to reject it. (Type II error or Beta-β)
- Which type of error is more serious depends on the situation at hand. In other words, the gravity of the error is context dependent.

- Here's an illustration of the four situations in a hypothesis test:

My Decision	The Truth: H_0 True	The Truth: H_0 False
Reject H_0	**Type I Error**	**Correct Decision**
Retain H_0	**Correct Decision**	**Type II Error**

Alpha is the probability of committing a Type I Error.

- Alpha is a conditional probability
- The probability of rejecting the null given that the null is in fact true

How often will a Type I error occur?

- Since a Type I error is rejecting a true null hypothesis (this is a mistake), the probability of a Type I error is our α level-we get to choose how often we are wrong

  Question: Why not choose a very small alpha level to minimize our chances of rejecting a true null and being guilty of committing a Type I Error?

  Answer: Because it becomes very difficult to correctly reject a false null which leads to a different type of error.

Beta is the probability of committing a Type II Error.

- Beta is a conditional probability
- The probability of failing to reject the null given that the null is in fact false

How often will a Type II error occur?

- Very difficult to answer because we do not know the actual population parameter
- As our alpha level increases our beta level decreases

Note: alpha plus beta do not equal one

- We can reduce Type II Error by increasing the sample size

Power of the test (1-β):

- The probability of correctly rejecting a False null hypothesis (this is correct)
- Power is a conditional probability:

  The probability of rejecting the null hypothesis given that it is false.
- We can increase the power of the test by increasing the sample size
- There is an inverse relationship between power and Type II Error
- If we increase our alpha level our power goes up

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Notes: Type I & Type II Errors

Type I Error-the probability of rejecting the null given that the null is true-this is measured by **alpha**.

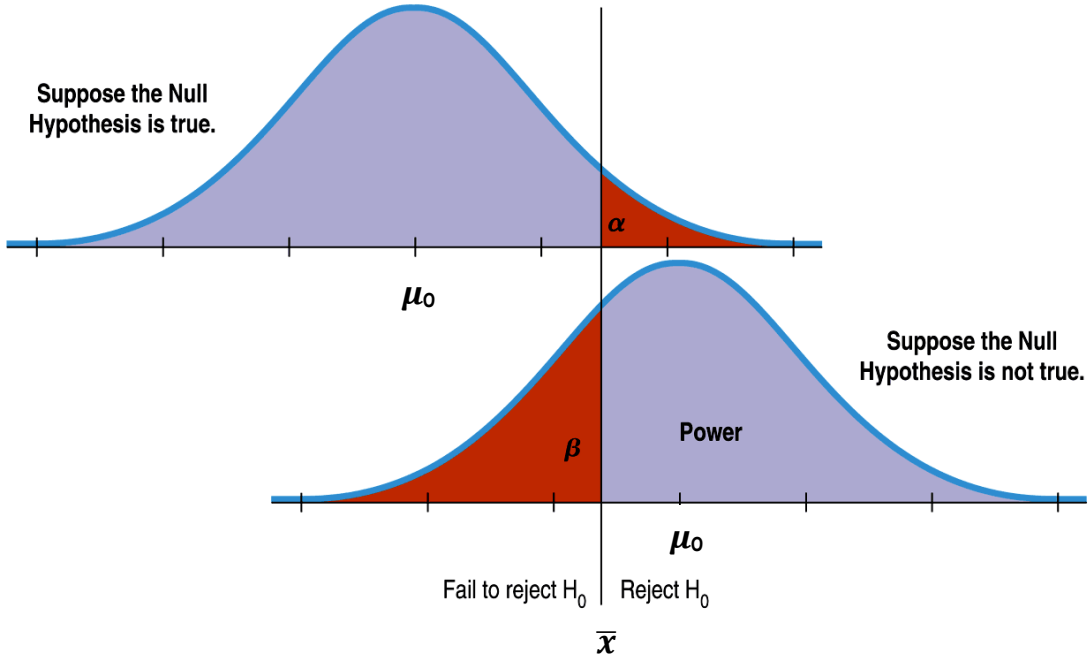
- A Type I Error cannot be committed if the null is not rejected
- A Type I Error may have been committed if the null is rejected (but only if the null is true)

Type II Error-the probability of accepting the null given that the null is in fact false-this is measured by **beta**.

- A Type II Error cannot be committed if the null is rejected
- A Type II Error may have been committed if the null is not rejected (but only if the null is false)

Power of the test $1-\beta$ represents the probability rejecting the null hypothesis given that it is false or correctly rejecting a false null.

The diagram below demonstrates the relationship between Type I and Type II Errors. Memorize and be able to draw this diagram



As alpha increases, Beta decrease and power increases

- More likely to reject the null hypothesis
- More likely to commit a Type I error.
- Less likely to commit a Type II error.

As alpha decrease, beta increases and Power decreases

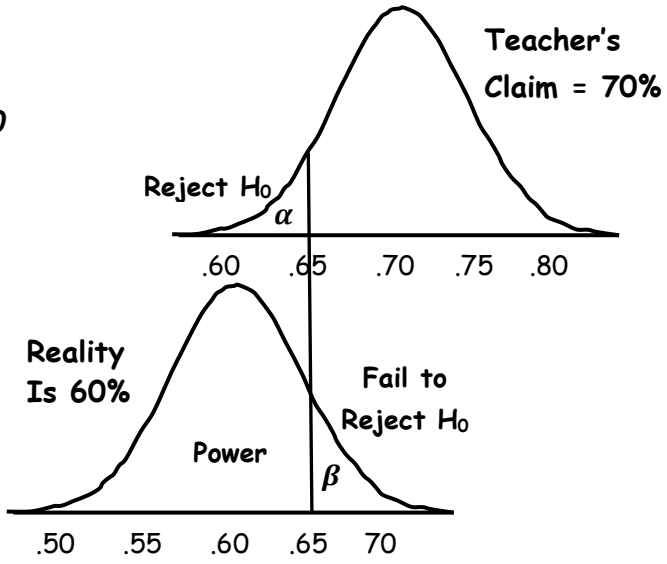
- Less likely to reject the null hypothesis
- Less likely to commit a Type I error.
- More likely to commit a Type II error.

Notes: Type I & Type II Errors

Example: A teacher claims that the test average is 70%, but you believe that the true figure is lower, you plan to gather an SRS and will reject the clam if in your sample the test average is 65% or less. The truth is that the test average is a 60%.

Null H_0: $\mu_0 = 70$

Alternative H_A: $\mu_0 < 70$



The upper graph shows the null hypothesis model with the claim that $\mu_0 = .70$ and the plan to reject the null if $\mu_0 < .65$. The lower graph shows the true model with $\mu_0 = .60$.

When will we fail to reject the null hypothesis that $\mu_0 = .70$?

When will we rightly conclude that the null hypothesis is incorrect?

Power gives the probability of avoiding a Type II error. **Power = $(1 - \beta)$**

Power has a different value for different possible correct values of the population parameter.

Choosing a smaller alpha results in a higher risk of committing a Type II error.

The greater the distance between the null and the true value of the population, the smaller the risk of committing a Type II error and the greater the power.

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Notes: Type I & Type II Errors

Scenario: Why were American's traveling from foreign countries tested before they re-entered the United States this Spring

Write the Hypothesis

Null

Alternative

Fill in the table		Actual Situation	
		The person has COVID-19	**The person does not Have COVID-19**
CDC Test Results	**The person has COVID-19**		
	The person does not Have COVID-19		

Answer the questions in context:

What is a Type I Error in this situation and what are the consequences:

What is a Type II Error in this situation and what are the consequences

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Recipe for Success: Type I Errors

Type I Error-The probability of Rejecting the null given that the null is True.

1. **Write the Hypothesis**
   - Null H_0:
   - Alternative H_A:

α = the probability of committing a Type I error
β = the probability of committing a Type II error
1- β = the power of the test

2. **Define parameter (μ or p) in context**

3. **Define a Type I Error α**

Definition: *The probability of rejecting the Null given that the Null is true.*

Remember: To commit a **Type I Error**, we must have **rejected the null** and were incorrect

simplified definition: *Rejecting a true Null and Accepting a False Alternative*

4. **Explain a Type I Error in Context of the problem**

In this case, the probability of rejecting_____ (*Restate* H_0) in favor of _____ (*Restate* H_A) given the fact_____ (*Restate* H_0) is true.

5. **Explain the consequences of Committing a Type I Error**

The consequences for committing at Type I Error are…

Explanation must be in the context and in simplified language.

Methods of Decreasing Type I Errors-α

In General:

As α ↓, power ↓, & β ↑

And

As α ↑, power ↑, & β ↓

1. **Decrease α – the level of significance**
   - Increases β-the probability of a Type II Error
   - More likely to accept a false null-*(this is an error)*
   - Power Decreases
2. **Decrease Power**
   - More likely to accept a false null—(Type II increases: negative)
   - Less likely to reject a true null—(Type I decreases: positive)
3. **Increase Sample Size**
   - Decreases Type I Error
   - Decreases Type II Error
   - Increases Power
   - (Costs money and Time)

P-value

1. **Write the Hypothesis**
   - Null H_0:
   - Alternative H_A:
2. **Define parameter (μ or p) in context**
3. **Define P-value**

P-value is the probability of getting a test statistic as extreme or more extreme given that the null is true.

4. **Explain P-value in the Context of the problem**

There is a _____ (*P-value*)% chance that we would get a test statistic this extreme in favor of_____ (*Restate* H_A) when in fact_____ (*Restate* H_0) is true

Notes: Type I & Type II Errors (Scenario)

Scenario: 2008 Form B Question 4. A researcher wants to conduct a study to test whether listening to soothing music for 20 minutes help to reduce diastolic blood pressure in patients with high blood pressure, compared to simply sitting quietly in a noise-free environment for 20 minutes. One hundred patients with high blood pressure at a large medical clinic are available to participate in this study.

(a) Propose a design for this study to compare these two treatments.

(b) The null hypothesis for this study is that there is no difference in the mean reduction of diastolic blood pressure for the two treatments and the alternative hypothesis is that the mean reduction in diastolic blood pressure is greater for the music treatment. If the null hypothesis is rejected, the clinic will offer this music therapy as a free service to their patients with high blood pressure. Describe a Type I error and the consequences in the context of this study

(c) A p-value of .0017 is calculated. Explain the meaning of the p-value in context and explain what type of error may have been committed.

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Recipe for Success: Type II Errors

Type II Error-The probability of Accepting the null given that the null is False.

1. **Write the Hypothesis**
   - Null H_0:
   - Alternative H_A:

α = the probability of committing a Type I error
β = the probability of committing a Type II error
1- β = the power of the test

2. **Define parameter (μ or p) in context**

3. **Define a Type II Error β**

Definition: The probability of accepting the Null given that it is false.

Remember: to commit a **Type II Error** we **failed to reject** the null and were incorrect.

simplified definition: Accepting a False Null and Rejecting a True Alternative

4. **Explain Type II Error in Context of the problem**

In this case, the probability of accepting_____given that
Restate H_0
_____is false and _____is true.
Restate H_0 *Restate* H_A

5. **Explain the consequences of Committing a Type II Error**

The consequences for committing a Type II Error are…

Explanation must be in the context and in simplified language.

Methods of Decreasing Type II Errors- β

As β ↓, power ↑, & α ↑

And

As β ↑, power ↓, & α ↓

1. **Increase α – the level of significance**
   - Increases-the probability of a Type I Error α
   - More likely to reject a True Null—*(this is an error)*
   - Power Increases
2. **Increase Power**
   - More likely to reject a true null—(Type I increases: negative)
   - Less likely to accept a false null—(Type II decreases: positive)
4. **Increase Sample Size**
   - Decreases Type I Error
   - Decreases Type II Error
   - Increases Power
   - (Costs money and Time)

P-value

1. **Write the Hypothesis**
   - Null H_0:
   - Alternative H_A:

2. **Define parameter (μ or p) in context**

3. **Define P-value**

P-value is the probability of getting a test statistic as extreme or more extreme given that the null is true.

4. **Explain P-value in the Context of the problem**

There is a _____% chance that we would get a test statistic
P-value
this extreme in favor of_____ when in fact_____is true.
Restate H_A *Restate* H_0

Notes: Type I & Type II Errors (Scenario)

Scenario: 2008 Form B Question 4. A researcher wants to conduct a study to test whether listening to soothing music for 20 minutes help to reduce diastolic blood pressure in patients with high blood pressure, compared to simply sitting quietly in a noise-free environment for 20 minutes. One hundred patients with high blood pressure at a large medical clinic are available to participate in this study.

(a) The null hypothesis for this study is that there is no difference in the mean reduction of diastolic blood pressure for the two treatments and the alternative hypothesis is that the mean reduction in diastolic blood pressure is greater for the music treatment. If the null hypothesis is rejected, the clinic will offer this music therapy as a free service to their patients with high blood pressure. Describe a Type II errors and its consequences in the context of this study, Also, discus in the context of this study which is more serious a Type I or Type II error.

A p-value of .247 is calculated. Explain the meaning of the p-value in context and explain what type of error may have been committed.

Multiple Choice Practice Errors

1. Which of the following statements is incorrect?
   (A) The significance level of a test is the probability of a Type II error.
   (B) Given a particular alternative, the power of a test against that alternative is 1 minus the probability of the Type II error associated with that alternative.
   (C) If the significance level remains fixed, increasing the sample size will reduce the probability of a Type II error.
   (D) If the significance level remains fixed, increasing the sample size will raise the power.
   (E) Holding the sample size fixed, increasing the significance level will decrease the probability of a Type II error.

2. A manufacturer of heart-lung machines periodically checks a sample of its product and performs a major recalibration if readings are sufficiently off target. Similarly, a rug factory periodically checks the sizes of its throw rugs coming off an assembly line and halts production if measurements are sufficiently off target. In both situations, we have the null hypothesis that the production equipment is performing satisfactorily. For each situation, which is the more serious concern, a Type I or Type II error?
   (A) Machine producer: Type I error, carpet manufacturer: Type I error
   (B) Machine producer: Type I error, carpet manufacturer: Type II error
   (C) Machine producer: Type II error, carpet manufacturer: Type I error
   (D) Machine producer: Type II error, carpet manufacturer: Type II error
   (E) This is impossible to answer without making an expected value judgment between human life and accurate throw rug sizes.

3. Which of following is incorrect?
   (A) The power of a test concerns its ability to detect a true alternative hypothesis.
   (B) The significance level of a test is the probability rejecting a true null hypothesis.
   (C) The probability of a Type I error plus the probability of a Type II error always equals 1.
   (D) Power equals 1 minus the probability of failing to reject a false null hypothesis.
   (E) Anything that makes a null hypothesis harder to reject will increase the probability of committing a Type II error.

4. Suppose H_0: $p = .6$, H_A: $p > .6$, and against the alternative $p = .7$, the power is .8. Which of the following is a valid conclusion?
   (A) The probability of committing a Type I error is .1.
   (B) If $p = .7$ is true, the probability of failing to reject H_0 is .2.
   (C) The probability of committing a Type II error is .3.
   (D) All of the above are valid conclusions.
   (E) None of the above are valid conclusions.

5. If all other variables remain constant, which of the following will not increase the power of a hypothesis test?
   (A) Increasing the sample size
   (B) Increasing the significance level
   (C) Increasing the probability of a Type II error
   (D) Decreased variability in the data
   (E) Increased distance between the true and the hypothesized parameter.

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Notes: Transforming & Combining Random Variables

Rules for Adding, Subtracting & Multiplying by Constants

Addition:

- Constants **can be** added to measures of center (mean & median)
- Constants **can NOT be** added to measures of spread (range, IQR, variance & standard deviation)

Subtraction:

- Constants **can be** subtracted to measures of center (mean & median)
- Constants **can NOT be** subtracted to measures of spread (range, IQR, variance & standard deviation)

Multiplication:

- Constants **can be** multiplied to measures of center (mean & median)
- Constants **can be** multiplied to measures of spread (range, standard deviation & IQR)
- Constants **must be squared** first and the result can be multiplied by a variance

Division:

- Measures of center (mean & median) **can be** divided by a constant
- Measures of spread (range, IQR & standard deviation) **can be** divided by the constant
- Variances can be divided by the square of the constant

Example:

Random Variable	Mean	Median	Range	IQR	Standard Deviation	Variance
(X)	23.5	21	15	6.3	2.7	7.29

Calculate the mean, median, range, IQR, standard deviation and variance for the following:

1. $X + 4$

Random Variable	Mean	Median	Range	IQR	Standard Deviation	Variance
(X)						

2. $X - 20$

Random Variable	Mean	Median	Range	IQR	Standard Deviation	Variance
(X)						

3. $3X$

Random Variable	Mean	Median	Range	IQR	Standard Deviation	Variance
(X)						

4. $X \div 4$

Random Variable	Mean	Median	Range	IQR	Standard Deviation	Variance
(X)						

5. $.8X + 2$

Random Variable	Mean	Median	Range	IQR	Standard Deviation	Variance
(X)						

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Notes: Transforming & Combining Random Variables

Random Variables: Rules for Combining Random Variables

Addition:

- **Means:** can be summed directly
- **Variances:** can be summed directly
- **Standard deviations: can NOT** be added. Must be converted to variances summed and then the square root of the sum must be taken to convert the summed variance to a standard deviation

Subtraction:

- **Means: can be** subtracted directly
- **Variances: can NOT** be subtracted and must be summed directly
- **Standard deviations: can NOT** be subtracted. Must be converted to variances summed and the square root of the sum must be taken to convert the summed variance to a standard deviation

Example:

Random Variable	Mean	Standard Deviation	Variance
(X)	23	4	
(Y)	29	5	

For the above independent Random Variables calculate the mean, standard deviation and variance for the following:

1. $X + Y$

Mean	Standard Deviation	Variance

2. $X - Y$

Mean	Standard Deviation	Variance

3. $.4X - .6Y$

Mean	Standard Deviation	Variance

4. $(X + X + X) \div 3$

Mean	Standard Deviation	Variance

5. $(Y + Y + Y + Y) \div 4$

Mean	Standard Deviation	Variance

6. $(X + X + X) \div 3 + (Y + Y + Y + Y) \div 4$

Mean	Standard Deviation	Variance

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Notes: Paired T-test Versus a 2-Sample T-test

Purpose: A 2-sample t-test is used to determine whether or not there exists a difference between the means of 2 independent groups.

When to Use: We often want to know how two groups differ, whether a treatment is better than a placebo control, or whether this year's results are better than last year's.

- **Paired T-tests: Samples are dependent a relationship exists**
  - Same subject tested before and after treatment is given
  - Same subject tested twice (receives both treatments)
  - Tests on Twins and the difference between each pair is computed
  - Tests on Spouses and the difference between each couple is computed
  - Tests on Siblings and the difference between each sibling pair is computed
  - Data groups must be the exact same length
- **2-samples T-tests: Samples are independent**
  - Treatments are randomly assigned and the average of one group is compared to another.
  - Data groups do not need to be the same length

Determining whether to use a Paired T-test or a 2-Sample T-test

1. An experiment measures people's lung capacity before and then after an exercise program to see if their fitness has improved.
   - Which T-test would you use? Paired T-test or independent 2 sample t-test
   - How many tails does this test have? 1-tailed or 2 tailed

2. A different experiment measures the lung capacity of one group who took one exercise program and another group who took a different exercise program to determine if a difference in outcomes existed.
   - Which T-test would you use? Paired T-test or independent 2 sample t-test
   - How many tails does this test have? 1-tailed or 2 tailed

3. You want to see if an educational program will help raise test scores. Your experimental hypothesis is "Post-training test scores will be significantly higher than Pre-training test scores."
   - Which T-test would you use? Paired T-test or independent 2 sample t-test
   - How many tails does this test have? 1-tailed or 2 tailed

4. For Each Scenario determine whether it would be more appropriate to use a 2-sample T-test or Paired T-test?

   Example Scenario
   - Comparing the average height of men and women
   - Comparing the weight of spouses
   - Comparing the BMI of teen agers to that adults
   - Comparing Cholesterol levels of a child to their parent
   - Comparing the blood pressures of patients receiving Drug A to those receiving Drug B
   - Comparing the weight of dieters before and after a diet.

Recipe for Success: 2-Sample T Hypothesis Test (difference of Means)

1. **Write the Hypothesis**
   - Null H_0: $\mu_1 = \mu_2$
   - Alternative H_A: $\mu_1 \neq \mu_2$ or $\mu_1 < \mu_2$ or $\mu_1 > \mu_2$
2. **Define μ_1 & μ_2** *in context*
3. **Write the Conditions**
   1. **Both samples are random**
   2. **n < 10% of the population**
   3. **Populations are independent**
   4. **Normal Population or *n > 30***

   If data is given, draw a boxplot or histogram-to show normality
4. **Write the Equation**

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

t = the number of standard deviations from the mean
μ_1 & μ_2 = the means of the population (may be assumed)
$\bar{x}_1$ & $\bar{x}_2$ = the means of the samples
s_1 & s_2 = the standard deviation of the sample
n_1 & n_2 = the size of the sample

5. **Enter Data** (if given)
   - Stat Edit
   - Enter Data in columns L_1 & L_2
   - 2nd Quit
   - Stat Calc
   - 1-Var Stats L_1: $\bar{x}_1$, s_1, n_1 and L_2: $\bar{x}_2$, s_2, n_2
6. **List & Label all of input values**

   $\bar{x}_1$, s_1, n_1, $\bar{x}_2$, s_2, n_2

   df (comes from the calculator)
   - Stat Tests
   - **4**: 2-SampTTest Enter
   - Highlight **Data** if data is used otherwise highlight **STATS**
   - **s_1 & s_2** comes from the problem or the data
   - **$\bar{x}_1$ & $\bar{x}_2$** comes from the problem or the data
   - **n_1 & n_2** comes from the problem or the data
   - **pooled** highlight no
   - **Choose ≠ or < or >** (look for key words)
7. **Plug values into the equation**
8. **Write the t and the p-value** The T and the p-value are calculated in step 5
9. **State the Decision**
   - The p-value is_____
   - compare to alpha: p-value (< or >) **alpha**
   - If the p-value is **less** than alpha, **Reject the Null**
   - If the p-value is **greater** than alpha, **Fail to reject the Null**
10. **Write the Conclusion**

Reject the Null: Our p-value is _____. We reject the Null. There is sufficient evidence at alpha = ___ to suggest that the difference in the true population mean for _____ is _____
Restate the definition of the 1st mean *Restate H_A ≠ or < or > 2nd mean definition*

Fail to Reject the Null: Our p-value is _____. We Fail to reject the Null. There is not sufficient evidence at alpha = _____ to suggest that the difference in true population mean for _____is _____
Restate the definition of the 1st mean *Restate H_A ≠ or < or > 2nd mean definition*

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Notes: 2-Sample T Hypothesis Test (Scenario)

Scenario 1: The average ACT score of an independent random sample of 16 non-athletes at Blinn College was compared to that of an independent random sample of 7 athletes. The school's guidance counselor was curious to know whether or not Blinn was admitting athletes with lower ACT scores than non-athletes. Perform an appropriate test and state your conclusion. Assume alpha =.05

Composite ACT Score		
Non-Athletes		Athletes
25	21	22
22	27	21
19	29	24
25	26	27
24	30	19
25	27	23
24	26	17
23	23	

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Recipe for Success: 2 Sample T-Confidence Intervals

1. **Define μ_1 & μ_2** *in context*
2. **Write the Conditions**
   1. **Both samples are random**
   2. **n < 10% of the population**
   3. **Populations are independent**
   4. **Normal Population or *n > 30***

   If data is given, draw a boxplot or histogram-to show normality
3. **Write the formula for the Test**

$$\bar{x}_1 - \bar{x}_2 \ \pm \ t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$\bar{x}_1$ & $\bar{x}_2$ = the means of the samples
s1 & s2 = the standard deviation of the sample
n1 & n2 = the size of the sample

4. **Graph and Shade**
5. **Enter the Data if Given**
   - Stat Edit
   - Enter Data in columns L_1 & L_2
   - 2nd Quit
   - Stat Calc
   - 1-Var Stats L_1: $\bar{x}_1$, s_1, n_1 and L_2: $\bar{x}_2$, s_2, n_2
6. **List & Label all of input values**
   $\bar{x}_1$, s1, n1, $\bar{x}_2$, s2, n2, df

   df (comes from the calculator)
   - Stat Tests
   - **0**:2-Samp T Int
   - Highlight **Data** if data is used otherwise highlight **STATS**
   - **s1 & s2** comes from the problem or the data
   - $\bar{x}_1$ & $\bar{x}_2$ comes from the problem or the data
   - **n1 & n2** comes from the problem or the data
   - **pooled** highlight no
7. **Calculate *t**
   - 2nd Vars
   - Inverse t
   - Area = $\frac{(1-Confidence\ level)}{2}$
   - **df** (comes from the calculator in the step above)
8. **Plug in the values**
9. **Write the interval**
10. **Write the Conclusion**

We are _____% confident that the true population mean difference for_____
Restate the definition of the μ_1
and _____ lies within the interval _____
Restate the definition of the μ_2

11. **Explain the meaning of the confidence level-***if asked*

In repeated sampling we expect this method to capture the true population mean difference for _____ and _____ _____% of the time.
Restate the definition of the μ_1 *Restate the definition of the* μ_2

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Notes: 2-Sample T Confidence Interval (Scenario)

Scenario 1: The average ACT score of an independent random sample of 16 non-athletes at Blinn College was compared to that of an independent random sample of 7 athletes. The school's guidance counselor was curious to know whether or not Blinn was admitting athletes with lower ACT scores than non-athletes. Create and interpret both the 95% confidence interval and confidence level. Is the 95% confidence interval consistent with your hypothesis test? Explain.

Composite ACT Score		
Non-Athletes		Athletes
25	21	22
22	27	21
19	29	24
25	26	27
24	30	19
25	27	23
24	26	17
23	23	

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Recipe for Success: 2-Sample T Hypothesis Test (difference of Means)

1. **Write the Hypothesis**
   - Null H_0: $\mu_1 = \mu_2$
   - Alternative H_A: $\mu_1 \neq \mu_2$ or $\mu_1 < \mu_2$ or $\mu_1 > \mu_2$
2. **Define μ_1 & μ_2** *in context*
3. **Write the Conditions**
   1. **Both samples are random**
   2. **n < 10% of the population**
   3. **Populations are independent**
   4. **Normal Population or *n > 30***

   If data is given, draw a boxplot or histogram-to show normality
4. **Write the Equation**

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

t = the number of standard deviations from the mean
μ_1 & μ_2 = the means of the population (may be assumed)
$\bar{x}_1$ & $\bar{x}_2$ = the means of the samples
s_1 & s_2 = the standard deviation of the sample
n_1 & n_2 = the size of the sample

5. **Enter Data** (if given)
   - Stat Edit
   - Enter Data in columns L_1 & L_2
   - 2nd Quit
   - Stat Calc
   - 1-Var Stats L_1: $\bar{x}_1$, s_1, n_1 and L_2: $\bar{x}_2$, s_2, n_2
6. **List & Label all of input values**

   $\bar{x}_1$, s_1, n_1, $\bar{x}_2$, s_2, n_2

   df (comes from the calculator)
   - Stat Tests
   - **4**: 2-SampTTest Enter
   - Highlight **Data** if data is used otherwise highlight **STATS**
   - **s_1 & s_2** comes from the problem or the data
   - **$\bar{x}_1$ & $\bar{x}_2$** comes from the problem or the data
   - **n_1 & n_2** comes from the problem or the data
   - **pooled** highlight no
   - **Choose ≠ or < or >** (look for key words)
7. **Plug values into the equation**
8. **Write the t and the p-value** The T and the p-value are calculated in step 5
9. **State the Decision**
   - The p-value is_____
   - compare to alpha: p-value (< or >) **alpha**
   - If the p-value is **less** than alpha, **Reject the Null**
   - If the p-value is **greater** than alpha, **Fail to reject the Null**
10. **Write the Conclusion**

Reject the Null: Our p-value is _____. We reject the Null. There is sufficient evidence at alpha = ___ to suggest that the difference in the true population mean for

_____ is _____

Restate the definition of the 1st mean *Restate H_A ≠ or < or > 2nd mean definition*

Fail to Reject the Null: Our p-value is _____. We Fail to reject the Null. There is not sufficient evidence at alpha = _____ to suggest that the difference in true population mean for

_____is _____

Restate the definition of the 1st mean *Restate H_A ≠ or < or > 2nd mean definition*

Notes: 2-Sample T Hypothesis Test (Scenario)

Scenario 2: In June 2002, the Journal of Applied Psychology reported to a study that examined whether the content of TV shows influenced the ability of viewers to recall brand names of items featured in the commercials. The researchers randomly assigned volunteers to watch one of three programs, each containing the same commercials. One of the programs had violent content, another sexual content, and the third neutral content. After the shows ended, the subjects were asked to recall the brands of products that were advertised. Results are summarized below:

Program Type

	Violent	Sexual	Neutral
Sample Size	**108**	**108**	**108**
Mean number of brands recalled	**2.08**	**1.71**	**3.17**
Standard Deviation	**1.87**	**1.76**	**1.77**

Is there a difference in ad memory between programs with neutral content and those with violent content? Test an appropriate hypothesis and state your conclusion. What decision should be made?

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Recipe for Success: 2 Sample T-Confidence Intervals

1. **Define μ_1 & μ_2** *in context*
2. **Write the Conditions**
   1. **Both samples are random**
   2. **n < 10% of the population**
   3. **Populations are independent**
   4. **Normal Population or *n > 30***

   If data is given, draw a boxplot or histogram-to show normality
3. **Write the formula for the Test**

$$\bar{x}_1 - \bar{x}_2 \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$\bar{x}_1$ & $\bar{x}_2$ = the means of the samples
s1 & s2 = the standard deviation of the sample
n1 & n2 = the size of the sample

4. **Graph and Shade**
5. **Enter the Data if Given**
   - Stat Edit
   - Enter Data in columns L_1 & L_2
   - 2nd Quit
   - Stat Calc
   - 1-Var Stats L_1: $\bar{x}_1$, s_1, n_1 and L_2: $\bar{x}_2$, s_2, n_2
6. **List & Label all of input values**
   $\bar{x}_1$, s1, n1, $\bar{x}_2$, s2, n2, df

   df (comes from the calculator)
   - Stat Tests
   - **0**:2-Samp T Int
   - Highlight **Data** if data is used otherwise highlight **STATS**
   - **s1 & s2** comes from the problem or the data
   - $\bar{x}_1$ & $\bar{x}_2$ comes from the problem or the data
   - **n1 & n2** comes from the problem or the data
   - **pooled** highlight no
7. **Calculate *t**
   - 2nd Vars
   - Inverse t
   - Area = $\frac{(1-Confidence\ level)}{2}$
   - **df** (comes from the calculator in the step above)
8. **Plug in the values**
9. **Write the interval**
10. **Write the Conclusion**

We are _____% confident that the true population mean difference for_____
Restate the definition of the μ_1
and _____ lies within the interval _____
Restate the definition of the μ_2

11. **Explain the meaning of the confidence level-***if asked*

In repeated sampling we expect this method to capture the true population mean difference for _____ and _____ _____% of the time.
Restate the definition of the μ_1 *Restate the definition of the* μ_2

Notes: 2-Sample T Confidence Interval (Scenario)

Scenario 2: In June 2002, the Journal of Applied Psychology reported to a study that examined whether the content of TV shows influenced the ability of viewers to recall brand names of items featured in the commercials. The researchers randomly assigned volunteers to watch one of three programs, each containing the same commercials. One of the programs had violent content, another sexual content, and the third neutral content. After the shows ended, the subjects were asked to recall the brands of products that were advertised. Results are summarized below:

Program Type

	Violent	Sexual	Neutral
Sample Size	**108**	**108**	**108**
Mean number of brands recalled	**2.08**	**1.71**	**3.17**
Standard Deviation	**1.87**	**1.76**	**1.77**

The company can afford to run ads during one TV show, and has decided not to sponsor a show with sexual content. Create a 95% confidence interval for the difference in the mean number of brand names remembered between groups watching neutral shows and those watching violent shows. What is your recommendation? Justify

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Recipe for Success: 2-Sample T Hypothesis Test (difference of Means)

1. **Write the Hypothesis**
   - Null H_0: $\mu_1 = \mu_2$
   - Alternative H_A: $\mu_1 \neq \mu_2$ or $\mu_1 < \mu_2$ or $\mu_1 > \mu_2$
2. **Define μ_1 & μ_2** *in context*
3. **Write the Conditions**
   1. **Both samples are random**
   2. **n < 10% of the population**
   3. **Populations are independent**
   4. **Normal Population or *n > 30***

   If data is given, draw a boxplot or histogram-to show normality
4. **Write the Equation**

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

t = the number of standard deviations from the mean

μ_1 & μ_2 = the means of the population (may be assumed)

$\bar{x}_1$ & $\bar{x}_2$ = the means of the samples

s_1 & s_2 = the standard deviation of the sample

n_1 & n_2 = the size of the sample

5. **Enter Data** (if given)
   - Stat Edit
   - Enter Data in columns L_1 & L_2
   - 2nd Quit
   - Stat Calc
   - 1-Var Stats L_1: $\bar{x}_1$, s_1, n_1 and L_2: $\bar{x}_2$, s_2, n_2
6. **List & Label all of input values**

   $\bar{x}_1$, s_1, n_1, $\bar{x}_2$, s_2, n_2

   df (comes from the calculator)
   - Stat Tests
   - **4**: 2-SampTTest Enter
   - Highlight **Data** if data is used otherwise highlight **STATS**
   - **s_1 & s_2** comes from the problem or the data
   - **$\bar{x}_1$ & $\bar{x}_2$** comes from the problem or the data
   - **n_1 & n_2** comes from the problem or the data
   - **pooled** highlight no
   - **Choose ≠ or < or >** (look for key words)
7. **Plug values into the equation**
8. **Write the t and the p-value** The T and the p-value are calculated in step 5
9. **State the Decision**
   - The p-value is_____
   - compare to alpha: p-value (< or >) **alpha**
   - If the p-value is **less** than alpha, **Reject the Null**
   - If the p-value is **greater** than alpha, **Fail to reject the Null**
10. **Write the Conclusion**

Reject the Null: Our p-value is _____. We reject the Null. There is sufficient evidence at alpha = ___ to suggest that the difference in the true population mean for

_____ **is** _____

Restate the definition of the 1st mean *Restate H_A ≠ or < or > 2nd mean definition*

Fail to Reject the Null: Our p-value is _____. We Fail to reject the Null. There is not sufficient evidence at alpha = _____ to suggest that the difference in true population mean for

_____**is** _____

Restate the definition of the 1st mean *Restate H_A ≠ or < or > 2nd mean definition*

Notes: 2-Sample T Hypothesis Test (Scenario)

Scenario 3: A statistics teacher wanted to know if males could navigate through a maze faster than females. A random sample of male and female Reagan students was selected and the times it took them to complete the maze were recorded below. Determine whether or not there exist a difference **between completion times for men and women**.

Maze Completion Times				
Female Times			Male Times	
120	335	185	285	285
88	172	131	251	76
147	217	277	220	54
160	178	140	94	285
223	264	177	75	77
80	230	121	180	257
79	108	364	204	112
256	205	94	188	119
97	180	125	88	103
85	122		176	
337	136			

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Recipe for Success: 2 Sample T-Confidence Intervals

1. **Define μ_1 & μ_2** *in context*
2. **Write the Conditions**
   1. **Both samples are random**
   2. **n < 10% of the population**
   3. **Populations are independent**
   4. **Normal Population or *n > 30***

   If data is given, draw a boxplot or histogram-to show normality
3. **Write the formula for the Test**

$$\bar{x}_1 - \bar{x}_2 \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$\bar{x}_1$ & $\bar{x}_2$ = the means of the samples
s_1 & s_2 = the standard deviation of the sample
n_1 & n_2 = the size of the sample

4. **Graph and Shade**
5. **Enter the Data if Given**
   - Stat Edit
   - Enter Data in columns L_1 & L_2
   - 2nd Quit
   - Stat Calc
   - 1-Var Stats L_1: $\bar{x}_1$, s_1, n_1 and L_2: $\bar{x}_2$, s_2, n_2
6. **List & Label all of input values**
   $\bar{x}_1$, s_1, n_1, $\bar{x}_2$, s_2, n_2, df

   df (comes from the calculator)
   - Stat Tests
   - **0**:2-Samp T Int
   - Highlight **Data** if data is used otherwise highlight **STATS**
   - **s_1 & s_2** comes from the problem or the data
   - **$\bar{x}_1$ & $\bar{x}_2$** comes from the problem or the data
   - **n_1 & n_2** comes from the problem or the data
   - **pooled** highlight no
7. **Calculate *t**
   - 2nd Vars
   - Inverse t
   - Area = $\frac{(1-Confidence\ level)}{2}$
   - **df** (comes from the calculator in the step above)
8. **Plug in the values**
9. **Write the interval**
10. **Write the Conclusion**

We are _____% confident that the true population mean difference for_____
Restate the definition of the ***μ_1***
and _____ lies within the interval _____
Restate the definition of the ***μ_2***

11. **Explain the meaning of the confidence level-***if asked*

In repeated sampling we expect this method to capture the true population mean difference for _____ and _____ _____% of the time.
Restate the definition of the ***μ_1*** *Restate the definition of the* ***μ_2***

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Notes: 2-Sample T Confidence Interval (Scenario)

Scenario 3: A statistics teacher wanted to know if males could navigate through a maze faster than females. A random sample of male and female Reagan students was selected and the times it took them to complete the maze were recorded below.

Create a 90% confidence interval for the scenario and explain your findings in context.

Maze Completion Times				
Female Times			**Male Times**	
120	**335**	**185**	**285**	**285**
88	**172**	**131**	**251**	**76**
147	**217**	**277**	**220**	**54**
160	**178**	**140**	**94**	**285**
223	**264**	**177**	**75**	**77**
80	**230**	**121**	**180**	**257**
79	**108**	**364**	**204**	**112**
256	**205**	**94**	**188**	**119**
97	**180**	**125**	**88**	**103**
85	**122**		**176**	
337	**136**			

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Notes: Recalling the Binomial Distribution

Bernoulli Trials: a random experiment with exactly two possible outcomes, "success" and "failure", in which the probability of success is the same every time the experiment is conducted.

4. two possible outcomes (success **"p"** and failure **"q"**).
5. the probability of success, **"p"**, is constant.
6. the trials are independent

Note: When we don't have an infinite population, the trials are not independent. However it is still okay to proceed as long as the sample is smaller than 10% of the population. **M&M example** (**n < 10%** of the population)

The Binomial Model: Gives us the probability for a random variable that counts the number of successes in a fixed number of Bernoulli trials.

Characteristics:

- Has 2 possible outcomes
- Each trial is independent
- the probability of success, **"p"**, is constant
- The trials are independent (subject to the note above)
- The number of trials is predetermined

Equation: $\sum_{x=0}^{n} \binom{n}{k} p^x q^{n-x} = \frac{n!}{x!(n-x)!} p^x q^{n-x}$

- **n** is the number of trials
- **x** is the number of success
- **p** is the probability or **proportion** of successes for the population
- **q (1-p)** the probability or **proportion** of failures for the population

Mean (μ) or E(X)=np **Variance or Var(X)** σ^2 **= npq** **Standard deviation** $\sigma = \sqrt{npq}$

Binomials are usually phrased as:

- What is the probability of some number of successes in a given number of trials? (Binomial PDF)
- What is the probability of no more than some number of successes in a given number of trials? (Binomial CDF)
- What is the probability of at least some number of successes in a given number of trials? (Binomial CDF) usually **1-**Binomial CDF

Scenario: Suppose that the probability of having a defective light bulb is .15

- Given a box of 30 lightbulbs what is the probability that 2 are defective?

- Given a box of 30 lightbulbs, what is the probability that no more than 2 are defective?

- Given a box of 30 lightbulbs, what is the probability that at least 28 are **not** defective?

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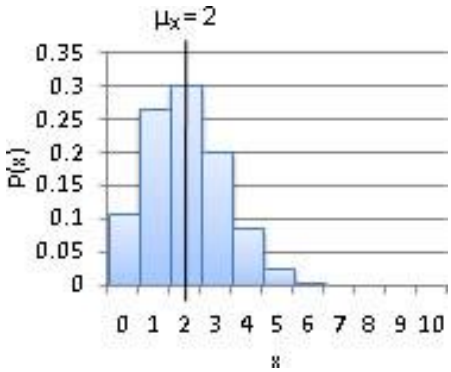
Notes: The Normal Approximation of the Binomial

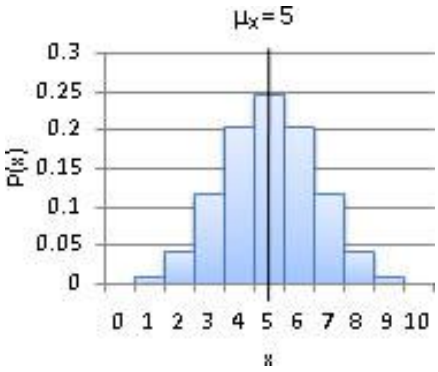
The best way to understand the effect of n and p on the shape of a binomial probability distribution is to look at some histograms, so let's look at some possibilities.

n=10, p=0.2



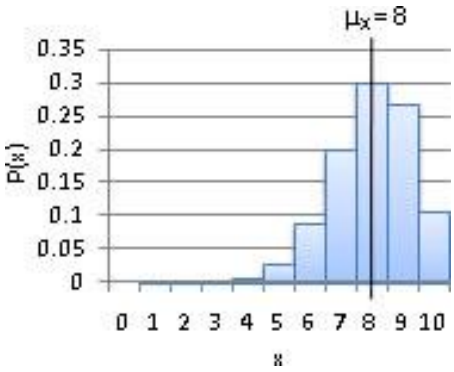
As P approaches 0 the binomial becomes progressively more skewed to the right.

n=10, p=0.5



As P approaches 0.5 the binomial becomes more normal

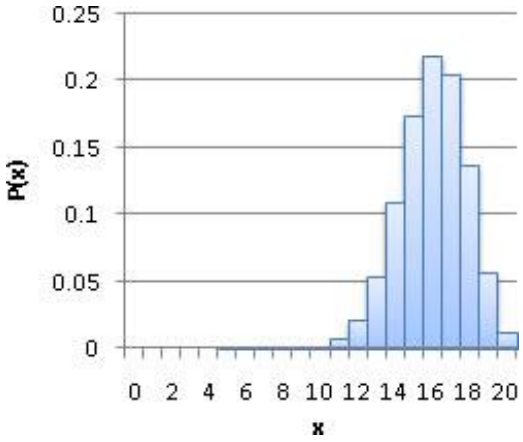
n=10, p=0.8



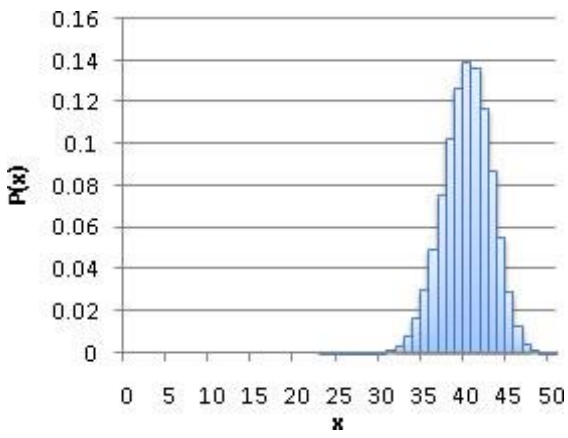
As P approaches 1 the binomial becomes progressively more skewed to the left.

Based on the above, it would appear that the distribution is symmetric only if p=0.5, but this isn't actually true. Watch what happens as the number of trials, n, increases:

n=20, p=0.8



n=50, p=0.8



Because of this property which is an essentially the application of the central limit theorem, we can use the normal model to approximate the binomial if the sample size is large enough.

Why would we do such a thing? The binomial becomes difficult to work with and can exceed the limits of our calculator if we have a substantial number trials.

Requirements: (sample size must be large enough)

1. **np ≥ 10** number of successes is greater than 10
2. **nq ≥ 10** number of failures is greater than 10

Scenario: An archer with an 80% bull's eye rate will be shooting 2000 arrows.

a) What are the mean and Standard deviation of the number of bull's eyes she might get?

b) Is a Normal Model appropriate here? Justify numerically

Notes: Distribution of Sample Proportions

Central Limit Theorem for Means-States that the sampling distribution of the expected values (means) of a population with a mean of μ and a standard deviation of σ will be approximately normal for any population regardless of the shape of the underlying population if the **sample is large enough**.

- **Distribution of Sample Means:** Follows a normal distribution where **E(x)** or $\mu = \overline{x}$ and $\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}}$ **or** $s_{\overline{x}} = \frac{s}{\sqrt{n}}$ when σ is unknown for any shaped distribution if the sample is large enough **n ≥ 30**

Central Limit Theorem for Proportions-States that the sampling distribution of the expected proportion **"P"** for a population with a standard deviation of σ will be approximately normal for any population regardless of the shape of the underlying population if the **sample is large enough**.

Conditions:

1. **Random Sample**
2. **n < 10% of the population** *(degree of dependence is minimal, relatively independent-remember we are sampling without replacement)*
3. **Sample is Large Enough** (necessary to claim normality by Central Limit Theorem)
   - $n\hat{p}$ > 10 where n is the sample size and $\hat{p}$ is the sample proportion
   - $n\hat{q}$ > 10 where n is the sample size and $\hat{q}$ is (1 - $\hat{p}$)

Distribution of Sample proportions: Follows a normal distribution with **z** = $\frac{\hat{p}-p}{\sqrt{\frac{(p)(q)}{n}}}$

- **The Center:**
  - **E(x)** or **"P"** is the population proportion
  - $\hat{P} = \frac{x}{n}$ **"P"** is the population proportion and $\hat{P}$ the sample proportion
    - **x** is the number of observed successes (outcomes of interest)
    - **n** is the sample size
- **The spread:**
  - $\sigma_p = \sqrt{\frac{(p)(q)}{n}}$ (standard error of the population proportion)-use for hypothesis test because we assume the claim of the null is true.
  - $\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}\hat{q}}{n}}$ (standard error of the sample proportion)-use for confidence intervals because we are looking for an estimate of the population proportion

Scenario: Assume that 27% of the students at Reagan wear contacts or other corrective eyewear and that we randomly select a sample of 110 students.

What is the distribution of $\hat{P}$? Specify the name of the distribution the mean and standard deviation and verify that all conditions are met.

B. What's the probability that more than 30% of the sample use corrective eyewear?

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Notes: Hypothesis Tests for 1-Sample Proportions

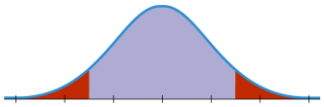
Hypothesis Tests: Remember the purpose of a hypothesis test is to determine whether the data support a claim or not. We always begin with the assumption that the null is true. Unless we have strong evidence against the null we retain or accept the null but we do not prove it. If we have strong evidence against the null we reject it in favor of the alternative. We do not prove the alternative.

- **We gather evidence against a claim**
- **We must judge plausibility of the evidence or data**
  - How likely are we to get data like this if the null really were true
  - We never have 100% certainty, but we can quantify our level belief in the evidence
  - The p-value tells us how surprised we are to see our results
- **We never prove a claim:** we are forced to accept the claim because we failed to reject it
- **We never prove the alternative:** we accept the alternative because we rejected the null and the alternative is our only remaining choice

The Hypothesis:

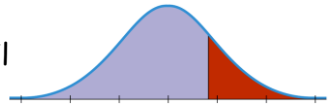
- The null- H_0: $P = P_0$ (P is the true population proportion & P_0 is the claimed population proportion)
- The alternative:
  - H_A: $P \neq P_0$ the true population is different than the claimed (2-tailed)

    Note: The alpha level is divided equally into both tails
    If our level of significance was 10%, 5% would be in each tail



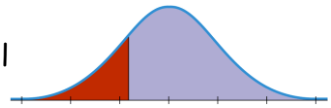
  - H_A: $P > P_0$ the true population proportion is greater than the claimed (1-tailed, upper)

    Note: The alpha level is in the upper tail
    If our level of significance was 10%, all of it would be in the Upper tail



  - H_A: $P < P_0$ the true population proportion is less than the claimed (1-tailed, lower)

    Note: The alpha level is in the upper tail
    If our level of significance was 10%, all of it would be in the Upper tail



P-Value: the probability we would find results as extreme or more extreme given that the null is true. How unlikely is too unlikely?

- **We decide based on the situation.**
- **Our threshold (alpha) should be determined prior to running our test.**

The formula: $$z = \frac{\hat{p} - p_0}{\sqrt{\frac{(p_0)(q_0)}{n}}}$$

Notice that this is a z-test. The reason this is a z-test is because P_0 is assumed to be the true population proportion and will continue to be so unless we find sufficient evidence to reject that claim.

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Recipe for Success: 1 Sample Proportion Hypothesis Test

1. **Write the Hypothesis**
   - Null H_0: $p_0 =$
   - Alternative H_A: $p_0 \neq$ or $<$ or $>$
2. **Define parameter p_0 *in context***
3. **Write the Conditions**
   - **Simple Random Sample**
   - **$n\hat{p} \geq 10$**
   - **$n\hat{q} \geq 10$**
   - **n is less than 10% of the population $\frac{n}{.1}$**
4. **Write the Equation**

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{(p_0)(q_0)}{n}}}$$

z = the number of standard deviations a value is from the center
p_0 = the population **proportion** or what is assumed to be true
$q_0 = 1 - p_0$ (q_0 = the expected proportion of failures)
n = the size of the sample
x = the number of successes
$\hat{p} = \frac{x}{n}$ sample proportion of successes-those that met criteria
$\hat{q} = 1 - \hat{p}$ the sample proportion of failures

5. **Draw the graph and Shade H_A**
6. **List & Label all of input values**
   - p_0 should be given
   - $q_0 = 1 - p_0$
   - x = the number of successes from the sample
   - n = the sample size
   - $\hat{p} = \frac{x}{n}$
   - $\hat{q} = 1 - \hat{p}$
7. **Plug values into the equation**
8. **Calculate the z and the p-value**
   - Stat Tests
   - 1-proportion z-test
   - p_0 comes from the problem
   - x comes from the problem or the data
   - **n** comes from the problem or the data
   - Choose $\neq$ or $<$ or $>$ *(using Shaded graph of H_A)*
9. **State the Decision**
   - The p-value is_____
   - If the p-value is less than alpha, Reject the Null
   - If the p-value is greater than alpha, Fail to reject the Null
10. **Write the Conclusion**

Reject the Null: Our p-value is _____. We reject the Null. There is sufficient evidence at alpha = ___ to suggest that the true population proportion for _____
Restate the definition of the p_0
is _____
Restate H_A $\neq$ or $<$ or $>$ p_0

Fail to Reject the Null: Our p-value is _____. We Fail to reject the Null. There is not sufficient evidence at alpha = ___ to suggest that the true population mean for
_____is _____
Restate the definition of the p_0 *Restate H_A $\neq$ or $<$ or $>$ p_0*

Notes: 1 Sample Proportions Hypothesis Test (Scenario)

Scenario 1: A union Spokesperson claims that 75% of union members will support a strike if their basic demands are not met. A company negotiator believes the true percentage is lower and runs a hypothesis test at the 10% significance level. What is the conclusion if 87 out or an SRS of 125 union members say they will strike?

Explain the meaning of the p-value in context and the type of Error that may have been committed.

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Notes: Confidence Intervals for 1-Sample Proportions

Confidence Intervals: Remember that the purpose of a confidence interval is to estimate a range of plausible values for the parameter of interest and follow the form:

estimate $\pm$ margin of error

- All confidence intervals are 2-tailed

  Note: alpha-- $\propto$ --is the combined area in the tails
- As we increase the level of confidence the margin of error (the interval width increases)
- As we increase alpha the margin of error & our confidence decreases (the interval width decreases)
- As we increase the sample size the margin of error decreases (the interval width decreases)
- As we increase the standard deviation the margin of error increases (the interval width increases)

Note: Because we only have sample data, we are never able to say what the exact population proportion or mean is. We can claim, with a certain level of confidence, that the population proportion or mean lies within a specified interval.

Confidence Intervals of proportions: $\hat{p} \pm z^* \sqrt{\frac{(\hat{p})(\hat{q})}{n}}$

- **Estimate:** The sample proportion "$\hat{P}$" is the estimate for the population proportion **"P"**
- **Margin of Error:** the maximum distance a sample statistic may lie from the true population parameter or vice versa
  - The **margin of error** is the (z-score)×(the standard error) or $\mathbf{z^*} \times \sqrt{\frac{\hat{p}\hat{q}}{n}}$
  - **Z-score**- the confidence level-provides the maximum number of standard of deviations that we believe the sample proportion may lie away from the true population proportion

    Note: The standard error is often referred to as the standard deviation
  - **standard error** $\sqrt{\frac{\hat{p}\hat{q}}{n}}$ the standard deviation of the sample proportion

Confidence Interval Claims

We Can Claim: ____% Confident that the true population proportion lies within the interval

We Can Claim: ____% of the intervals collected by this method will capture the population proportion

Note: We are making a claim about the population proportion and not the sample proportion. We do not need to make a claim about the sample proportion. The sample proportion will be in the center of the interval 100% of the time. The sample proportion is the estimate.

We Cannot Claim:

- ____% of the values lie within the interval—*the interval may be wrong*
- ____% chance that a randomly selected ____will lie within the interval—*the interval may be wrong*
- ____% of samples will result in this interval-*the interval may be wrong*
- ____% probability that the true population proportion lies within the interval

(do not use probability; use confidence)

Remember: The purpose of the interval is to provide a range of values for the parameter it is not designed to capture a given percentage of the data and as the sample size increases the amount of the population that lies with the interval decreases.

Note: If the claim (the null) lies within the interval, there is not enough evidence to reject the null. The result is not significant.

Confidence Intervals

Confidence Interval Hints: Please remember that although the population does not change every sample is going to be different with a different mean and a different standard deviation. As a consequence every confidence interval will have a different point estimator ($\bar{x}$ *or* $\hat{p}$) and different end points. We do not have any confidence in the endpoints, we are confident in the method and believe with a given level of confidence that we captured the true population parameter.

We only make claims about the population. We never make claims about the sample because we know all of the values of the sample. We know the exact center and spread of the sample, but we use those values to make inferences/claims about the population:

1. To determine the average spent on entertainment during a year in college, a simple random sample of 35 students is interviewed, showing a man of $825 with a standard deviation of $240. Which of the following is the best interpretation of a 90 percent confidence interval estimate for the average spent on entertainment during a year in college?

   (A) 90 percent of college students spend between $756 and $894 on entertainment yearly.
   (B) 90 percent of college students spend a mean dollar amount on entertainment yearly that is between $756 and $894.
   (C) We are 90 percent confident that college students spend between $756 and $894 on entertainment yearly.
   (D) We are 90 percent confident that college students spend a mean dollar amount between $756 and $894 on entertainment yearly.
   (E) We are 90 percent confident that in the chosen sample, the mean dollar amount spent on entertainment yearly by college students is between $756 and $894.

2. A planning board in Elm County is interested in estimating the proportion of its residents that are in favor of offering incentives to high-tech industries to build plants in that county. A random sample of Elm County residents was selected. All of the selected residents were asked, "Are you in favor of offering incentives to high-tech industries to build plants in your county?" A 95 percent confidence interval for the proportion of residents in favor of offering incentives was calculated to be 0.54 ± 0.05. Which of the following statements is correct?

   (A) At the 95% confidence level, the estimate of 0.54 is within 0.05 of the true proportion of county residents in favor of offering incentives to high-tech industries to build plants in the county.

   (B) At the 95% confidence level, the majority of area residents are in favor of offering incentives to high-tech industries to build plants in the county.

   (C) In repeated sampling, 95% of sample proportions will fall in the interval (0.49, 0.59)

   (D) In repeated sampling, the true proportion of county residents in favor of offering incentives to high-tech industries to build plants in the county will fall in the interval (0.49, 0.59).

   (E) In repeated sampling, 95% of the time the true proportion of county residents in favor of offering incentives to high-tech industries to build plants in the county will be equal to 0.54.

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Recipe for Success: 1 Sample Proportions (Confidence Interval)

1. **Define parameter p_0** *(the population proportion)* **in context**
2. **Write the Conditions**
   - **Simple Random Sample**
   - **$n\hat{p} \geq 10$**
   - **$n\hat{q} \geq 10$**
   - **n is less than 10% of the population $\frac{n}{.1}$**
3. **Write the Equation**

$$\hat{p} \pm z^* \sqrt{\frac{(\hat{p})(\hat{q})}{n}}$$

4. **List the Values**

   $\hat{p} = \frac{x}{n}$ sample proportion of successes-those that met criteria
   $\hat{q} = 1 - \hat{p}$ the sample proportion of failures
   z*= the number of standard deviations a value is from the center
   x = the number of successes or measured outcomes of interest
   n = the size of the sample

5. **Calculate z***
   - 2nd Vars
   - Inverse Norm
   - Area = $\frac{(1-Confidence\ level)}{2}$
   - $\mu = 0$ and $\sigma = 1$
6. **Plug in the values**
7. **Calculate the Interval**
   - Stat Tests
   - **1-PropZInt**
   - **x** comes from the problem or the data
   - **n** comes from the problem or the data
   - **C-Level** Confidence level comes from the problem
8. **Write the interval**
9. **Write the Conclusion**

   We are _____% confident that the true population proportion for _____ lies within the interval _____.

   Restate the definition of the p_0

10. **Explain the meaning of the confidence level-***if asked*

    In repeated sampling, we expect that this method will capture the true population proportion _____percent of the time.

    Restate the Confidence Level

Notes: 1-Sample Proportion Confidence Intervals (Scenarios)

Scenario 1: The local news has taken a SRS of union members and found that 87 out of 125 union members surveyed said that they will strike. Calculate a 95% confidence level and provide a conclusion.

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Recipe for Success: 1 Sample Proportion Hypothesis Test

1. **Write the Hypothesis**
   - Null H_0: p_0 =
   - Alternative H_A: $p_0 \neq$ or $<$ or $>$
2. **Define parameter p_0 *in context***
3. **Write the Conditions**
   - **Simple Random Sample**
   - **$n\hat{p} \geq 10$**
   - **$n\hat{q} \geq 10$**
   - **n is less than 10% of the population $\frac{n}{.1}$**
4. **Write the Equation**

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{(p_0)(q_0)}{n}}}$$

z = the number of standard deviations a value is from the center
p_0 = the population **proportion** or what is assumed to be true
q_0 = **1 - p_0** (q_0 = the expected proportion of failures)
n = the size of the sample
x = the number of successes
$\hat{p} = \frac{x}{n}$ sample proportion of successes-those that met criteria
$\hat{q}$ = **1 - $\hat{p}$** the sample proportion of failures

5. **Draw the graph and Shade H_A**
6. **List & Label all of input values**
   - p_0 should be given
   - q_0 = **1 - p_0**
   - ***x*** = the number of successes from the sample
   - ***n*** = the sample size
   - $\hat{p} = \frac{x}{n}$
7. **Plug values into the equation**
   - $\hat{q}$ = **1 - $\hat{p}$**
8. **Calculate the z and the p-value**
   - Stat Tests
   - 1-proportion z-test
   - p_0 comes from the problem
   - ***x*** comes from the problem or the data
   - **n** comes from the problem or the data
   - Choose $\neq$ or $<$ or $>$ *(using Shaded graph of H_A)*
9. **State the Decision**
   - The p-value is_____
   - If the p-value is less than alpha, Reject the Null
   - If the p-value is greater than alpha, Fail to reject the Null
10. **Write the Conclusion**

Reject the Null: Our p-value is _____. We reject the Null. There is sufficient evidence at alpha = ___ to suggest that the true population proportion for _____
Restate the definition of the p_0
is _____
Restate H_A $\neq$ or $<$ or $>$ p_0

Fail to Reject the Null: Our p-value is _____. We Fail to reject the Null. There is not sufficient evidence at alpha = ___ to suggest that the true population mean for
_____is _____
Restate the definition of the p_0 *Restate H_A $\neq$ or $<$ or $>$ p_0*

Notes: 1 Sample Proportions Hypothesis Test (Scenario)

Scenario 2: For a particular shopping mall, 60% of the visitors spend over twenty-five dollars when they visit the mall. The mall was recently modeled and the shops are trying to determine if the percent of visitors spending at least twenty-five dollars has changed. A SRS of 550 people leaving the mall showed that 64% had spent over $25.00. Provide statistical evidence to answer the vendor's question using a hypothesis test.

Explain the meaning of the p-value in context and the type of Error that may have been committed.

Notes: 1 Sample Proportions Hypothesis Test (Scenario)

Recipe for Success: 1 Sample Proportions (Confidence Interval)

1. **Define parameter p_0** *(the population proportion)* **in context**
2. **Write the Conditions**
   - **Simple Random Sample**
   - **$n\hat{p} \geq 10$**
   - **$n\hat{q} \geq 10$**
   - **n is less than 10% of the population $\frac{n}{.1}$**
3. **Write the Equation**

$$\hat{p} \pm z^* \sqrt{\frac{(\hat{p})(\hat{q})}{n}}$$

4. **List the Values**

$\hat{p} = \frac{x}{n}$ sample proportion of successes-those that met criteria
$\hat{q} = 1 - \hat{p}$ the sample proportion of failures
z* = the number of standard deviations a value is from the center
x = the number of successes or measured outcomes of interest
n = the size of the sample

5. **Calculate z***
   - 2nd Vars
   - Inverse Norm
   - Area = $\frac{(1-Confidence\ level)}{2}$
   - $\mu = 0$ and $\sigma = 1$
6. **Plug in the values**
7. **Calculate the Interval**
   - Stat Tests
   - **1-PropZInt**
   - **x** comes from the problem or the data
   - **n** comes from the problem or the data
   - **C-Level** Confidence level comes from the problem
8. **Write the interval**
9. **Write the Conclusion**

We are _____% confident that the true population proportion for _____ lies within the interval _____.

Restate the definition of the p_0

10. **Explain the meaning of the confidence level-***if asked*

In repeated sampling, we expect that this method will capture the true population proportion _____percent of the time.

Restate the Confidence Level

Notes: 1-Sample Proportion Confidence Intervals (Scenarios)

Scenario 2: A SRS of 550 people leaving the mall showed that 64% had spent over $25.00. Calculate a 99% confidence level and provide a conclusion.

Notes: 1 Sample Proportions Hypothesis Test (Scenario)

Scenario 3: 1998 Question 5. A large university provides housing for 10 percent of its graduate students to live on campus. The university's housing office thinks that the percentage of graduate students looking for housing campus may be more than 10 percent. The housing office decides to survey a random sample of graduate students, and 62 of the 481 respondents say that they are looking for housing on campus.

(a) One the basis of the survey data, would you recommend that the housing office consider increasing the amount of housing on campus available to graduate students? Give appropriate evidence to support your recommendation.
In addition to the 481 graduate students who responded to the survey, there were 19 who did not respond. If these 19 had responded, is it possible that your recommendation would have changed? Explain.

Since we have already rejected, we need to find out if we would have failed to reject if all 19 had not been interested in on-campus housing.

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Sample Size Calculations

Given the confidence level and the standard deviation of the population, the sample size that will produce a predetermined margin of error ***ME*** of the confidence interval estimate of **P** *the true population proportion* is given by $\mathbf{n} = \frac{(z^*)^2 \hat{p}\hat{q}}{ME^2}$

Note: You can memorize this or you can use the confidence interval equation & solve using basic algebra.

1. An experiment finds that 27% of 53 subjects report improvement after using a new medicine. What sample size is necessary to create a margin of error of plus or minus 3% with a 98% confidence level?

What sample size is necessary to estimate the outcome of an election with a margin of error of plus or minus 3% with 95% confidence? **Hint: What information am I missing** and yes, we can do this problem by making an assumption.

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Sample Size of Confidence Intervals

1. For a given large sample size, which of the following gives the smallest margin of error in calculating a confidence interval for a population proportion?

   (A) 90 percent confidence with $\hat{p}$ = .15
   (B) 95 percent confidence with $\hat{p}$ = .15
   (C) 99 percent confidence with $\hat{p}$ = .15
   (D) 90 percent confidence with $\hat{p}$ = .23
   (E) 95 percent confidence with $\hat{p}$ = .23

2. Two 95 percent confidence interval estimates are obtained: 1st (78.5, 84.5) and 2nd (80.3, 88.2).

   a. If the sample sizes are the same, which has the larger standard deviation?
   b. If the sample standard deviations are the same, which has the larger sample size?

   (A) a. 1st b. 1st
   (B) a. 1st b. 2nd
   (C) a. 2nd b. 1st
   (D) a. 2nd b. 2nd
   (E) More information is needed to answer these questions.

6. Which of the following would result in the widest confidence interval?

   (A) Small sample size and 95 percent confidence
   (B) Small sample size and 99 percent confidence
   (C) Large sample size and 95 percent confidence
   (D) Large sample size and 99 percent confidence
   (E) This cannot be answered without knowing an appropriate standard deviation

3. In a New York Times poll measuring a candidate's popularity, the newspaper claimed that in 19 of 20 cases its poll results should be no more than three percentage points off in either direction. What confidence level are pollsters working with, and what size sample should they have obtained?

   (A) 3%, 20
   (B) 6%, 20
   (C) 6%, 100
   (D) 95%, 33
   (E) 95%, 1068

4. A *t*-statistic was used to conduct a test of the null hypothesis H_0: $\mu = 0$ against the alternative H_a: $\mu \neq 0$. The *p*-value was 0.056. A two-sided confidence interval for μ is to be constructed. Of the following, which is the largest level of confidence for which the confidence interval will NOT contain 0?

   (A) 90% confidence
   (B) 93% confidence
   (C) 95% confidence
   (D) 98% confidence
   (E) 99% confidence

Sample Size of Confidence Intervals

5. The school superintendent wants to know what percentage of property owners are willing to support an increase in school taxes. What size sample should be obtained to determine with 90 percent confidence the support level to within 5 percent?

   (A) 17
   (B) 33
   (C) 271
   (D) 289
   (E) 1,083

6. A political action group wishes to learn the government approval rating on the environment. From a past study, they know that they will have to poll 270 people for their desired level of confidence. If they want to keep the same confidence interval and have a margin of error one third the size, how many people will they have to poll?

   (A) 30
   (B) 90
   (C) 468
   (D) 810
   (E) 2,430

7. In a random survey of 450 adults, 28 percent said that they felt their credit card debt is too high. With what degree of confidence can the pollster say that 28 ± 4 percent of adults believe that their credit card debt is too high?

   (A) 70.0 percent
   (B) 91.0 percent
   (C) 94.1 percent
   (D) 95.0 percent
   (E) 96.0 percent

8. A guidance counselor wishes to determine the mean number of changes in academic major by college students to within $\pm$ 0.1 at a 90 percent confidence level. What sample size should be chosen if it is known that the standard deviation is 0.45?

   (A) 8
   (B) 54
   (C) 55
   (D) 78
   (E) 110

Sample Size of Confidence Intervals

9. Two confidence interval estimates from the same sample are (72.2, 77.8) and (71.3, 78.7). One estimate is at the 95 percent level, and the other is at the 99 percent level. Which is which?
   (A) (72.2, 77.8) is the 95 percent level.
   (B) (72.2, 77.8) is the 99 percent level.
   (C) This question cannot be answered without knowing the sample size.
   (D) This question cannot be answered without knowing the sample standard deviation
   (E) This question cannot be answered without knowing both the sample size and standard deviation.

10. In general, how does doubling the sample size change the confidence interval size?

   (A) Doubles the interval size
   (B) Halves the interval size
   (C) Multiplies the interval size by 1.414
   (D) Divides the interval size by 1.414
   (E) This question cannot be answered without knowing the sample size.

11. A 90 percent confidence interval is to be created to estimate the proportion of television viewers in a certain area who favor moving the broadcast of the late weeknight news to an hour earlier than it is currently. Initially, the confidence interval will be created using a sample of 9,000 viewers in the area. Assuming that the sample proportion does not change, what would be the relationship between the width of the original confidence interval that is created based on a sample of only 1,000 viewers in the area?

   (A) The second confidence interval would be 9 times as wide as the original confidence interval.

   (B) The second confidence interval would be 3 times as wide as the original confidence interval.

   (C) The width of the second confidence interval would be equal to the width of the original confidence interval.

   (D) The second confidence interval would be $\frac{1}{3}$ times as wide as the original confidence interval.

   (E) The second confidence interval would be $\frac{1}{9}$ times as wide as the original confidence interval.

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Notes: 2-Sample Proportion Confidence Intervals

Purpose: A 2-sample z-interval for proportions is used to create a plausible range for the difference between the proportions of 2 independent populations.

Confidence Intervals: estimate ± margin of error

- **Estimate:** The difference between 2 sample proportions.
- **Margin of error:** For a given confidence level, the maximum distance a sample statistic may lie from the true population parameter and equals z^* ×(Standard error)
  - **z*** the maximum number of standard deviations for a given confidence level
  - **Standard error** - the standard deviation of the combined variables
- **Formula:** $\hat{p}_1 - \hat{p}_2 \pm z^* \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$ **estimate ± margin of error.**

  Remember: variances and standard deviations must be independent in order to combine. The standard deviation of a 2-sample proportion is $\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$ and the variance is $\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}$

Confidence Interval Claims

We Can Claim: ____% Confident that the true population proportion lies within the interval

We Can Claim: ____% of the intervals collected by this method will capture the population proportion

Note: We are making a claim about the population proportion and not the sample proportion. We do not need to make a claim about the sample proportion. The sample proportion will be in the center of the interval 100% of the time. The sample proportion is the estimate.

We Cannot Claim:

- ____% of the values lie within the interval—*the interval may be wrong*
- ____% chance that a randomly selected ____will lie within the interval—*the interval may be wrong*
- ____% of samples will result in this interval-*the interval may be wrong*
- ____% probability that the true population proportion lies within the interval

(do not use probability; use confidence)

Remember: The purpose of the interval is to provide a range of values for the parameter it is not designed to capture a given percentage of the data and as the sample size increases the amount of the population that lies with the interval decreases.

Note: If the claim (the null) lies within the interval, there is not enough evidence to reject the null. The result is not significant.

Changes in Interval width

- All confidence intervals are 2-tailed **Note:** alpha-- $\propto$ --is the combined area in the tails
- As we increase the level of confidence the margin of error (the interval width increases)
- As we increase alpha the margin of error & our confidence decreases (the interval width decreases)
- As we increase the sample size the margin of error decreases (the interval width decreases)
- As we increase the standard deviation the margin of error increases (the interval width increases)

Note: Because we only have sample data, we are never able to say what the exact difference in population proportion or means is. We can claim with a certain level of confidence that the population proportion or mean difference lies within a specified interval.

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Recipe for Success: Confidence Interval-2 Sample Proportions

1. **Define p_1 & p_2 in context** *(the population proportions)*

   p_1 = the population **proportion** for the 1st proportion
   P_2 = the population **proportion** for the 2nd proportion

2. **Write the Conditions** *(must be for both sets of data)*

   - **Independent Random Samples**
   - **$n\hat{p} \geq 10$**
   - **$n\hat{q} \geq 10$**
   - **n is less than 10% of the population $\frac{n}{.1}$**

3. **Write the Equation**

$$\hat{p}_1 - \hat{p}_2 \pm z^* \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

4. **List the Values**

   z = the number of standard deviations a value is from the center
   n_1 = the size of the sample of the 1st proportion
   x_1 = the number of outcomes of interest of the 1st proportion
   $\hat{p}_1 = \frac{x_1}{n_1}$ 1st sample proportion of interest
   n_2 = the size of the sample of the 2nd proportion
   x_2 = the number of outcomes of interest of the 2nd proportion
   $\hat{p}_2 = \frac{x_2}{n_2}$ 2nd sample proportion of interest

5. **Calculate z***

   - 2nd Vars
   - Inverse Norm
   - Area = $\frac{(1 - Confidence\ level)}{2}$
   - $\mu = 0$ and $\sigma = 1$

6. **Plug in the values**

7. **Calculate the Interval**

   - Stat Tests
   - **2-PropZInt**
   - x_1 comes from the problem or the data
   - n_1 comes from the problem or the data
   - x_2 comes from the problem or the data
   - n_2 comes from the problem or the data
   - **C-Level** Confidence level comes from the problem

8. **Write the interval**

9. **Write the Conclusion**

   We are _____% confident that the true population proportion difference between _____ and _____ lies within the interval _____.

   Restate the definition of the p_1 *Restate the definition of the p_2*

10. **Determining significance.**

    - **If 0 is in the interval**—There is **No** significant difference
    - **If 0 is not in the interval-**There **Is** a significant difference

Notes: Confidence Intervals-2 Sample Proportions

Scenario 1: A grocery store manager notes in an SRS of 85 people going through the express line that only 10 paid with checks. In an SRS of 92 customers passing through the regular line 37 paid with checks. Construct a 95% confidence interval for the difference.

Notes: Confidence Intervals-2 Sample Proportions

Scenario 2: 84% of an SRS of 125 nurses express job satisfaction working 7am-3pm shifts, while 72% of a SRS of 150 nurses express satisfaction working 11pm-7am shifts. Establish a 90% confidence interval for the difference

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Notes: 2-Sample Proportion Hypothesis Tests

Purpose: A 2-sample hypothesis test for proportions is used to determine whether there is a difference between the proportions of 2 independent populations.

Pooling

The Null hypothesis is H_0: $p_1 = p_2$ or H_0: $p_1 - p_2 = 0$ **indicating a claim of no difference**

We believe there is no difference between the two proportions for each population. The standard deviation of a 2 sample proportion is $\sqrt{\frac{\hat{p}_c\hat{q}_c}{n_1} + \frac{\hat{p}_c\hat{q}_c}{n_2}}$ and is determined by the proportion.

Because we believe that both proportions are equal and because standard deviation is solely determined by the proportion it follows that the standard deviations of both populations must be equal.

Consequently we combine or pool the data counts to get one overall proportion using the formula:

$$\hat{p}_c = \frac{x_1+x_2}{n_1+n_2} \text{ or } \hat{p}_p = \frac{x_1+x_2}{n_1+n_2}$$

We only pool for 2-sample proportion hypothesis tests.

We do NOT pool for 2-sample confidence intervals for proportions

We do NOT pool for test of means

We do NOT pool for confidence intervals for means.

We only pool for 2-sample proportion hypothesis tests.

Formula for the test statistic: $z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{(\hat{p}_c)(\hat{q}_c)}{n_1} + \frac{(\hat{p}_c)(\hat{q}_c)}{n_2}}}$

Where: $\hat{p}_c = \frac{x_1+x_2}{n_1+n_2}$

z = the number of standard deviations a value is from the center

n_1 = the size of the sample of the 1st proportion

x_1 = the number of outcomes of interest of the 1st proportion

$\hat{p}_1 = \frac{x_1}{n_1}$ 1st sample proportion of interest

n_2 = the size of the sample of the 2nd proportion

x_2 = the number of outcomes of interest of the 2nd proportion

$\hat{p}_2 = \frac{x_2}{n_2}$ 2nd sample proportion of interest

$\hat{p}_c$ = the 2 combined or pooled proportion successes

$\hat{q}_c = 1 - \hat{p}_c$ the 2 combined or pooled proportion successes

Conditions: Both intervals and hypothesis tests for 2 sample proportions are based on the following conditions that must be listed and checked:

- Simple Random Sample or Random Assignment
- $n\hat{p} \geq 10$ & $n\hat{q} \geq 10$ Making certain the sample is large enough to apply the central limit theorem
- n is less than 10% of the population $\frac{n}{.1}$ Making certain the probability does not change substantially.

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Recipe for Success: Hypothesis Test: 2 Sample Proportions

1. **Write your Hypothesis**
   - Null H_0: $p_1 = p_2$
   - Alternative H_A: $p_1 \neq$ or $<$ or $>$ p_2
2. **Define p_1 & p_2 *in context***
   - p_1 = the population **proportion** for the 1st proportion
   - P_2 = the population **proportion** for the 2nd proportion
3. **Write the Conditions**
   (must be for both sets of data)
   - **Independent Random Samples**
   - **n is less than 10% of the population** $\frac{n}{.1}$
   - $n\hat{p} \geq 10$
   - $n\hat{q} \geq 10$
4. **Write the Equations**

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{(\hat{p}_c)(\hat{q}_c)}{n_1} + \frac{(\hat{p}_c)(\hat{q}_c)}{n_2}}}$$

$$\hat{p}_c = \frac{x_1 + x_2}{n_1 + n_2}$$

z = the number of standard deviations a value is from the center
n_1 = the size of the sample of the 1st proportion
x_1 = the number of outcomes of interest of the 1st proportion
$\hat{p}_1 = \frac{x_1}{n_1}$ 1st sample proportion of interest
n_2 = the size of the sample of the 2nd proportion
x_2 = the number of outcomes of interest of the 2nd proportion
$\hat{p}_2 = \frac{x_2}{n_2}$ 2nd sample proportion of interest
$\hat{p}_c$ = the 2 combined or pooled proportion successes
$\hat{q}_c = 1 - \hat{p}_c$ the 2 combined or pooled proportion successes

5. **List & Label all of input values**
   Calculate $\hat{p}_1$ & $\hat{p}_2$ & $\hat{p}_c$ & $\hat{q}_c$
6. **Plug values into the equation**
7. **Calculate the z and the p-value**
   - Stat Tests
   - 2-proportion z-test
   - x_1 & x_2 comes from the problem or the data
   - n_1 & n_2 comes from the problem or the data
   - Choose $\neq$ or $<$ or $>$
8. **State the Decision**
   - The p-value is_____
   - If the p-value is less than alpha, Reject the Null
   - If the p-value is greater than alpha, Fail to reject the Null
9. **Write the Conclusion**

Reject the Null: Our p-value is _____. We reject the Null. There is sufficient evidence at alpha = _____ to suggest that the true population proportion _____ is
Restate the definition of the p_1

_____ **true population proportion** _____.
Restate H_A $\neq$ or $<$ or $>$ *Restate the definition of the p_2*

Fail to Reject the Null: Our p-value is _____. We Fail to reject the Null. There is not sufficient evidence at alpha = _____ to suggest that the true population proportion for
_____is _____ than the true population proportion for
Restate the definition of the p_1 *Restate H_A $\neq$ or $<$ or $>$*

_____.
Restate the definition of the p_2

Notes: Hypothesis Tests-2 Sample Proportions (Scenario)

Scenario 1: Suppose that in an early election campaign a telephone poll of 800 registered voters shows 460 in favor of a particular candidate. Just before the election, a second poll shows only 520 of 1,000 registered voters expressing the same preference. At the 10% significance level is there sufficient evidence to suggest that the candidate's popularity has decreased?

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Recipe for Success: Hypothesis Test: 2 Sample Proportions

1. **Write your Hypothesis**
   - Null H_0: $p_1 = p_2$
   - Alternative H_A: $p_1 \neq$ or $<$ or $>$ p_2
2. **Define p_1 & p_2 in context**
   - p_1 = the population **proportion** for the 1st proportion
   - P_2 = the population **proportion** for the 2nd proportion
3. **Write the Conditions** *(must be for both sets of data)*
   - **Independent Random Samples**
   - **n is less than 10% of the population** $\frac{n}{.1}$
   - $n\hat{p} \geq 10$
   - $n\hat{q} \geq 10$
4. **Write the Equations**

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{(\hat{p}_c)(\hat{q}_c)}{n_1} + \frac{(\hat{p}_c)(\hat{q}_c)}{n_2}}}$$

$$\hat{p}_c = \frac{x_1 + x_2}{n_1 + n_2}$$

z = the number of standard deviations a value is from the center
n_1 = the size of the sample of the 1st proportion
x_1 = the number of outcomes of interest of the 1st proportion
$\hat{p}_1 = \frac{x_1}{n_1}$ 1st sample proportion of interest
n_2 = the size of the sample of the 2nd proportion
x_2 = the number of outcomes of interest of the 2nd proportion
$\hat{p}_2 = \frac{x_2}{n_2}$ 2nd sample proportion of interest
$\hat{p}_c$ = the 2 combined or pooled proportion successes
$\hat{q}_c = 1 - \hat{p}_c$ the 2 combined or pooled proportion successes

5. **List & Label all of input values**
   Calculate $\hat{p}_1$ & $\hat{p}_2$ & $\hat{p}_c$ & $\hat{q}_c$
6. **Plug values into the equation**
7. **Calculate the z and the p-value**
   - Stat Tests
   - 2-proportion z-test
   - x_1 & x_2 comes from the problem or the data
   - n_1 & n_2 comes from the problem or the data
   - Choose $\neq$ or $<$ or $>$
8. **State the Decision**
   - The p-value is_____
   - If the p-value is less than alpha, Reject the Null
   - If the p-value is greater than alpha, Fail to reject the Null
9. **Write the Conclusion**

Reject the Null: Our p-value is _____. We reject the Null. There is sufficient evidence at alpha = _____ to suggest that the true population proportion _____ is
Restate the definition of the p_1

_____ **true population proportion** _____.
Restate H_A $\neq$ or $<$ or $>$ *Restate the definition of the p_2*

Fail to Reject the Null: Our p-value is _____. We Fail to reject the Null. There is not sufficient evidence at alpha = _____ to suggest that the true population proportion for

_____is _____ than the true population proportion for
Restate the definition of the p_1 *Restate H_A $\neq$ or $<$ or $>$*

_____.
Restate the definition of the p_2

Notes: Hypothesis Tests-2 Sample Proportions (Scenario)

Scenario 2: An automobile manufacturer randomly assigns cars to 2 distinct assembly procedures. In a sample of 350 cars coming off the line using the 1st procedure there are 28 with major defects, while a sample of 500 autos from the 2nd shows 32 defects. Is this significant at the 10% level?

Notes: Hypothesis Tests-2 Sample Proportions (Scenario)

Scenario 3: 2016 Question 5: A polling agency showed the following two statements to a random sample of 1,048 adults in the United States.

Environment Statement: Protection of the environment should be given priority over economic growth.

Economy Statement: Economic growth should be given priority over protection of the environment.

The order in which the statements were shown was randomly selected for each person in the sample. After reading the statements, each person was asked to choose the statement that was most consistent with his or her opinion. The results are shown in the table.

	Environment Statement	Economy Statement	No Preference
Percent of Sample	58%	37%	5%

(A) Assume the conditions for inference have been met. Construct and interpret a 95% confidence interval for the proportion of all adults in the United States who would have chosen the economy statement.

(B) One of the conditions for inference that was met is that the number who chose the economy statement and the number who did not choose the economy statement are both greater than 10. Explain why it is necessary to satisfy that condition.

A suggestion was made to use a two-sample Z-interval for a difference between proportions to investigate whether the difference in proportions between adults in United States who would have chosen the environment state and adults in the United States who would have chosen the economy statement is statistically significant. Is the two-sample z-interval for a difference between proportions an appropriate procedure to investigate the difference? Justify your answer.

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Notes: Scatterplots

Previously we talked about how this course is divided into 4 large sections:

1. **The Collection** of Data (methods and cautions)
2. **The Display & Description** of the Data that was collected
3. **The Probability** of Obtaining the Results we observed
4. **Making Inferences or Decisions** based on the Likelihood of the results we obtained

When we were studying the display and description of data, there was one type of data that we avoided discussing in any detail and that is bivariate data. Our primary focus was on univariate data. For a Univariate graph one axis (typically the x) listed all possible outcome/response of the variable of interest while the y-axis kept track of the counts ie. the number of times a particular x-variable outcome/response occurred.

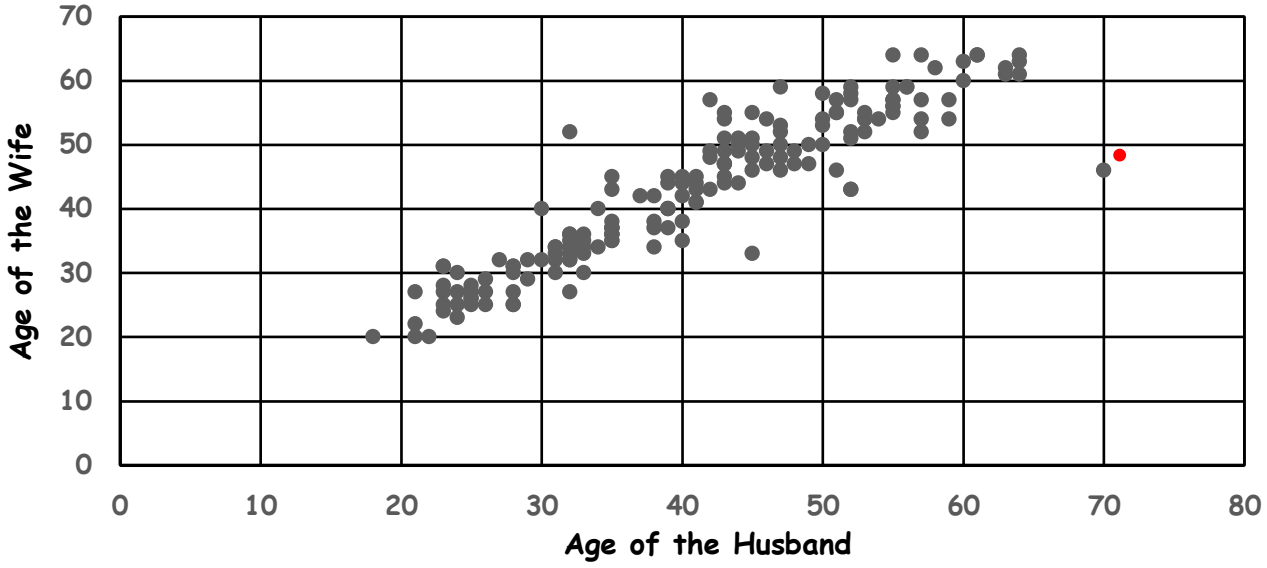
While both variables must be quantitative, bivariate data does not keep track of counts. Instead bivariate data provides the distinct measurements of a response variable (y) for a given value of the explanatory variable (x). The paired measurement is plotted as a single point on a scatterplot.

- Explanatory variable—the x variable that is believed to be the variable that drives the relationship (inputs)
- Response variable-the y variable which is the result of a change in the x-variable (outputs)

The plotting of multiple responses on a scatterplot allow us to determine what type of relationship exists between the two variables. We are able to determine how strong of a relationship exists and how predictive a pattern may be.

Scatterplots allow us to see the strength or lack thereof of the relationship/association between 2 variables. They also allow us to see patterns and unusual features such as clusters, gaps, outliers and points of influence.

Example: Husband's age versus wife's age. In general, the older the husband is, the older the wife is. But is the relationship perfect? Can we make some predictions?



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Notes: Creating Scatterplots

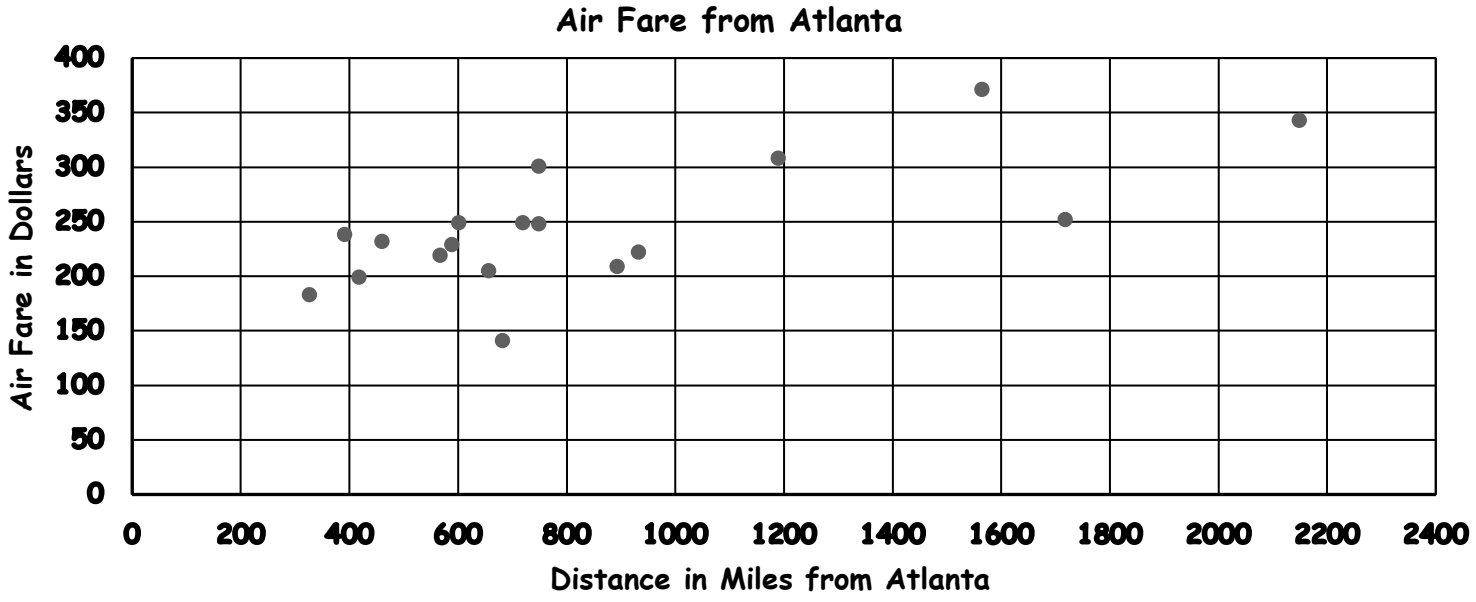
1. **Turn on Stat Diagnostics**
   - Press **MODE**
   - ↓ **STATDIAGNOSTICS:**
   - → Highlight **ON**
   - Press **ENTER**
   - Press **2nd Mode/Quit**

2. **Input the Data**
   - Press **STAT**
   - Highlight **EDIT**
   - Press **ENTER**
   - Enter "**x**" values in **L_1**
   - Enter "**y**" values in **L_2**
   - Press **2nd Mode/Quit**

3. **Graphing: Scatter Plot**
   - Press **2nd STAT PLOT**
   - Highlight **1: Plot 1** Press ENTER
   - Highlight **On** Press ENTER
   - ↓ Highlight First Graph Press ENTER
   - ↓ XList: Press **2nd L_1** Enter
   - ↓ YList: Press **2nd L_2** Enter
   - Press **ZOOM 9**

Atlanta to:	Distance L1	Fare L2
Baltimore	568	219
Boston	933	222
Dallas	720	249
Denver	1190	308
Detroit	602	249
Kansas City	683	141
Las Vegas	1719	252
Miami	589	229
Memphis	327	183
Minneapolis	894	209
New Orleans	419	199
New York	749	248
Oklahoma City	749	301
Orlando	392	238
Philadelphia	657	205
St. Louis	461	232
Salt Lake	1565	371
Seattle	2150	343

Summary Statistics		
Atlanta to:	Distance L1	Fare L2
Means	853.7	244.33
Standard Deviation	497.8	56.37



Notes: Linear Regression

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Notes: Linear Regression

Linear regression -an attempt to model the relationship between two variables by fitting a linear equation to observed data. (the line is often referred to as the line of best fit) One variable is considered to be an explanatory variable, and the other is considered to be a dependent variable. In our example graph the explanatory variable (x) is the distance in miles from Atlanta and our dependent variable (y) is the fare.

The algorithm for creating the line of best fit is designed to minimize the sum of the overall actual distances from the predictive line of best fit.

The linear regression equation: $\hat{y} = \beta_0 + \beta_1 x + \varepsilon$ which is essentially **y = b + mx** or **y = mx + b**

$\hat{y}$ is the value that the model predicts for y

β_0 is the value of the y-intercept of the regression equation which is the value of the equation when x is zero

β_1 is the slope of the regression line (units of y per units of x)

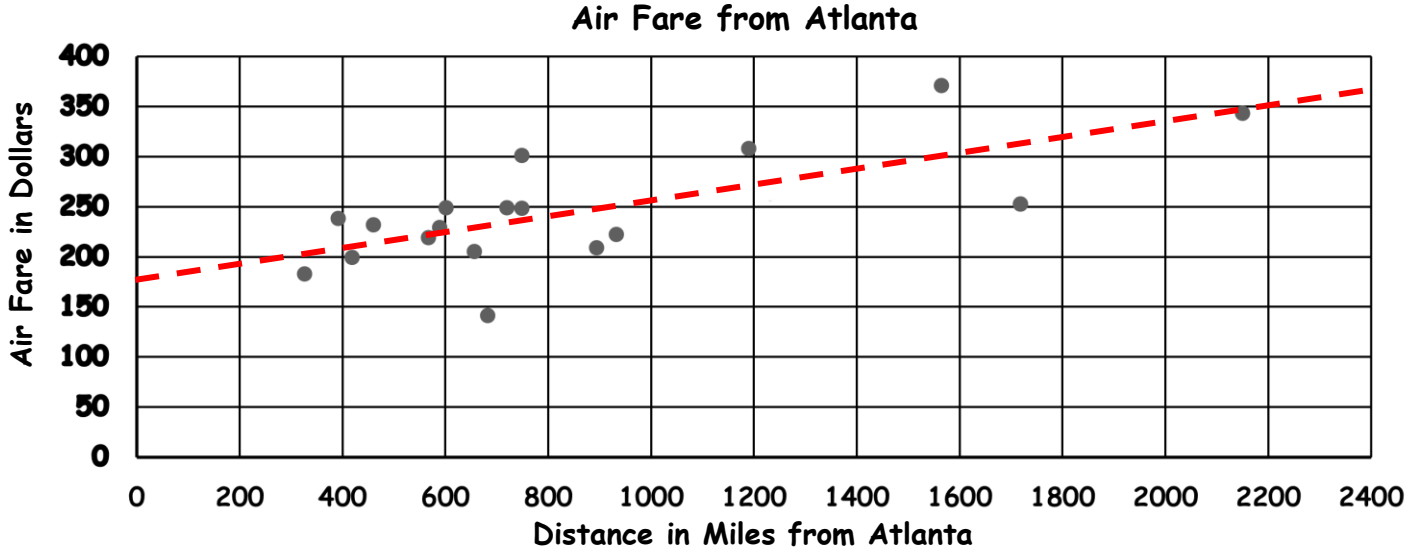
ε is the **error term or residual**. It is how far an actual value is from a predicted value and is found by subtracting the **observed - expected** or $y - \hat{y}$. The sum of the errors for a regression line is zero.

1. **Calculate the Regression Statistics**
   - Regression Equation **y = a + bx**
   - Slope: $B_1 = b$
   - Y-intercept: $B_0 = a$
   - Correlation Coefficient: **r**
   - Coefficient of Determination: r^2

   - Press **STAT**
   - →Highlight ***CALC***
   - ↓ **LinReg (a + bx)**
   - ↓ XList: Press **2nd** L_1 Enter
   - ↓ YList: Press **2nd** L_2 Enter
   - ↓ Store RegEQ: Press **2nd ALPHA TRACE** ENTER
   - Press **2nd Mode/Quit**

2. **Graphing the Regression Equation**
   - Press **Zoom 9**

B_0 = a=177.21, B_1=b=.0786 $\hat{y} = 177.21 + .0786x$ **r^2= .4820** **r= .6943**



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Notes: Residuals

As much as we would like there to be a perfect linear model with all of the data points situated on the predictive line that almost always never happens. The vertical distance the actual point is from the predictive line is known as the error term or residual.

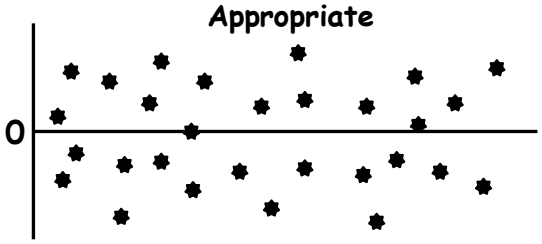
ε is the **error term or residual**. It is how far an actual value is from a predicted value and is found by subtracting the **observed - expected** or $y - \hat{y}$ or **actual - predicted**

Residual = Actual value - Model Value
Error = Data - Predicted

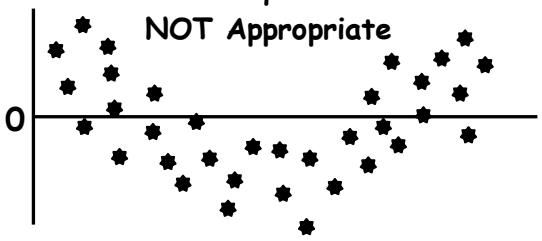
Actual value = Model value + Residual
Data = Predicted + Error

The sum of the errors for a regression line is zero.
For a linear equation to be appropriate, the graph of the Errors/Residuals must be random and without a pattern on a residual plot.

Linear Equation Is Appropriate



Linear Equation Is NOT Appropriate



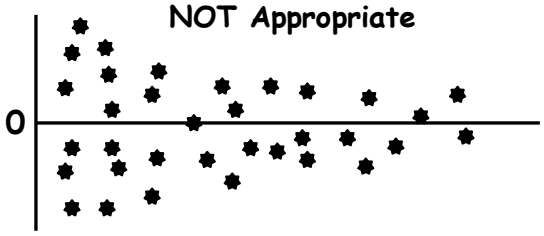
- **Calculating Residuals** *(actual – predicted)*
  - Press **STAT**
  - Highlight **EDIT;** Press **ENTER**
  - → Column L_3 ↑ Highlight L_3
  - Press **2nd STAT/LIST**
  - ↓ Highlight **7 RESID**
  - Press ENTER twice

  The residuals are in L_3

A **positive** residual means the **actual is greater** than the predicted-**above the regression line**

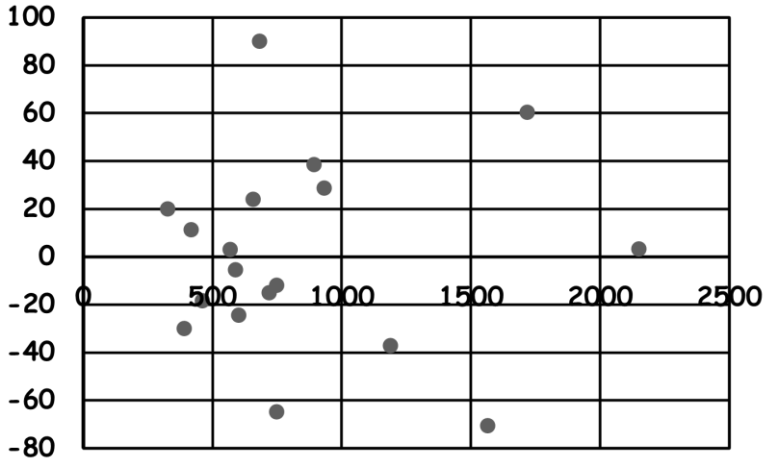
A **negative** residual means the **actual is less** than the predicted-**below the regression line**

Linear Equation Is NOT Appropriate



2. **Calculate the Standard Deviation of the Residuals "S"**
   - Press **STAT**→Highlight ***CALC***
   - ↓ **1-VAR STATS** Press **2nd 3** (L_3)
   - ↓ ↓ Press **Enter**
   - Write the values for **Σx^2** & **n**
   - Divide: **Σx^2** by **(n-2)** so $\frac{\Sigma x^2}{(n-2)}$
   - Press **^ .5** so $\sqrt{\frac{\sum x^2}{(n-2)}}$

Residual Plot for Airfare vs. Distance



3. **Graphing Residuals** *(actual – predicted)*
   - Press **2nd STAT PLOT**
   - Highlight **1: Plot 1** Press ENTER
   - Highlight **On** Press ENTER
   - ↓ Highlight First Graph Press ENTER
   - ↓ XList: Press **2nd L_1** Enter
   - ↓ YList: Press **2nd L_3** Enter
   - Press **ZOOM 9**

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Notes: Correlation

The residual plot may indicate that a line is appropriate, but it does not tell us how strong the linear relationship is between the response and explanatory variable. In order to determine the strength of a linear relationship we calculate **"r" the correlation coefficient**. 5

Note: we only talk about a correlation coefficient when we are discussing linear relationships.

When we calculate the regression Statistics, the calculator automatically give us "r" assuming STATS Diagnostics is turned on.

The **correlation coefficient** measures how closely the points in a scatter diagram are spread around the regression line. The value of the correlation coefficient always lies in the range of -1 to 1 that is, $-1 \le \rho \le 1$ and $-1 \le r \le 1$ where ρ is the correlation coefficient calculated for the population data and r is the correlation coefficient calculated for the sample data.

If ***r* = 1** (*perfect positive linear correlation*) The slope is positive and the data points lie directly on the line of regression.

If ***r* = -1** (*perfect negative linear correlation*) The slope is negative and the data points lie directly on the line of regression.

If ***r* is close to zero**; (*there is no **Linear** correlation between the two variables*). However the variables may have a very high correlation.

Rules of Thumb:

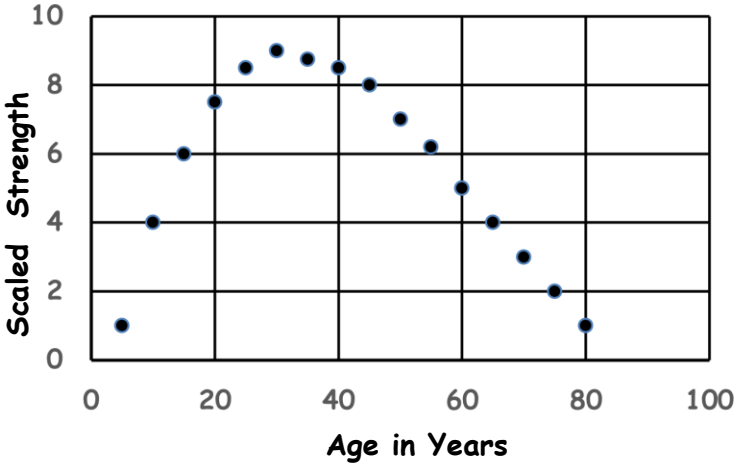
If $|r| \ge .75$ there is a strong linear association

If $.40 < |r| < .75$ there is a moderate linear association

If $|r| < .40$ there is a weak linear association

Caution: *A high correlation or association does not mean causation*

Note: Just because r is low does not mean that the variables are not highly correlated. Consider the following example graphs.



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Recipe for Success: Definition Template for Regression

Slope β_1 = m = $\frac{\Delta y}{\Delta x}$ (using y = m x + b notation)

_____ is the expected amount of change in _____
Value of the slope *Restate the definition of Y*

for a 1 unit change in _____ .
Restate the definition of x

Slope Intercept β_0 = b = constant (using y = m x + b notation)

The amount of _____ that you begin with or would have if
Restate the definition of Y
the amount of_____ = zero.
Restate the definition of x

Correlation Coefficient (r) = $\sqrt{R^2}$

- **$|r| \geq .75$** There is a strong _____linear relationship between
(+ or −)--*use the sign of the slope*
_____ and _____.
Restate the definition of Y *Restate the definition of x*

$.40 < |r| < .75$ There is a moderately strong _____linear
(+ or −)--*use the sign of the slope*
relationship between _____and_____.
Restate the definition of Y *Restate the definition of x*

- **$|r| < .40$** There is a weak _____linear relationship between
(+ or −)--*use the sign of the slope*
_____ and _____.
Restate the definition of Y *Restate the definition of x*

Coefficient of Determination (R^2)

_____% of the variation in the _____ can be explained by
Restate the definition of Y
changes in the_____
Restate the definition of x

S The **standard deviation of the residuals** is _____ and measures the variance in

_____ for a given amount of _____.
Restate the definition of Y *Restate the definition of x*

Standard Error of the Slope: The standard error of the slope is_____. Because the slope is estimated from the sample, other samples are likely to have differing slopes. The standard error of the slope quantifies the amount of variation in sample slopes that could be expected from different samples.

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Notes: Linear Regression Definitions

Atlanta to:	Distance L1	Fare L2
Baltimore	568	219
Boston	933	222
Dallas	720	249
Denver	1190	308
Detroit	602	249
Kansas City	683	141
Las Vegas	1719	252
Miami	589	229
Memphis	327	183
Minneapolis	894	209
New Orleans	419	199
New York	749	248
Oklahoma City	749	301
Orlando	392	238
Philadelphia	657	205
St. Louis	461	232
Salt Lake	1565	371
Seattle	2150	343

Summary Statistics		
Atlanta to:	**Distance L1**	**Fare L2**
Means	853.7	244.33
Standard Deviation	497.8	56.37

1. Define x:

2. Define y:

3. Write the equation of the linear model.

4. Find the slope of the regression line and explain what the slope means in this context.

5. Find the *y*-intercept of the regression line and explain what the *y*-intercept means in this context.

6. Calculate "S" and explain what it means in context.

7. Find *r* and explain and discuss its strength.

8. Find r^2 Explain what r^2 means in this context.

9. Estimate the fare for a 750-mile flight.

10. Estimate the fare for a 392-mile flight.

11. Based on your answer in **"10"** calculate the residual for a 392 mile flight.

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Recipe for Success: Hypothesis Test for Linear Regression (using data)

1. **Define X & Y in context**

2. **Write your Hypothesis**
   - Null H_0: $\beta_1 = 0$
   - Alternative H_A: $\beta_1 \neq 0$

   H_0: There is no linear relationship between x & y

   H_a: There is a linear relationship between x & y

3. **Define β_1 *in context***

   $\frac{\Delta y}{\Delta x}$ = the change in y for every 1 unit change in x

4. **Write the Conditions**
   - Random Sample
   - Linear Scatterplot
   - No pattern in residual plot
   - Distribution of the residuals is normal

5. **Write the Equations**

   $t = \frac{b}{s_b}$ $s_b = \frac{\sqrt{\frac{\sum(y-\hat{y})^2}{n-2}}}{\sqrt{\sum(x-\bar{x})^2}}$

   β_1 = the population slope of the regression line
   t = the number of standard deviations from the mean
   b = the slope of the regression line from the sample
   s_b = the standard error of the regression line
   n = the sample size
   df = n-2

6. **Enter Data**
   - **Stat Edit**
   - Enter X into L_1
   - Enter Y into L_2

7. **Calculate the p-value**
   - **Stat Tests**
   - LinRegTTest→Enter
   - X List- L_1
   - Y List- L_2
   - Freq 1
   - $\beta_1 \neq$ *(or whatever your hypothesis is)*
   - *ReqEQ Alpha Trace*

8. **State the Decision**
   - The p-value is____
   - If the p-value is less than alpha, Reject the Null
   - If the p-value is greater than alpha, Fail to reject the Null

9. **Write the Conclusion**

Reject the Null: Our p-value is _____. We reject the Null. There is sufficient evidence at alpha = _____ to suggest that the true population slope is **NOT** zero. There is a linear relationship between_____ and _____.
Restate the definition of x *Restate the definition of y.*

Fail to Reject the Null: Our p-value is _____. We Fail to reject the Null. There is not sufficient evidence at alpha = _____ to suggest that the true population slope is different than zero. No linear relationship exists between_____ and _____.
Restate the definition of y. *Restate the definition of x*

Notes: Hypothesis Test for the Slope of the Regression Line (Data Scenario)

Scenario 1: Using the Data from the air fare and distance scenario, perform a hypothesis test with an alpha level of 0.05 to determine whether or not the slope is significant.

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Recipe for Success: Confidence Intervals for Linear Regression (using data)

1. **Define X & Y in context**

2. **Define β_1 in context**

   $\frac{\Delta y}{\Delta x}$ = the change in y for every 1 unit change in x

3. **Write the Conditions**

   - Random Sample
   - Linear Scatterplot
   - No pattern in residual plot
   - Distribution of the residuals is normal

3. **Write the formula for the Test**

   $$b \pm t^* s_b$$

   $$s_b = \frac{\sqrt{\frac{\sum(y-\hat{y})^2}{n-2}}}{\sqrt{\sum(x-\bar{x})^2}}$$

   β_1 = the population slope of the regression line
   t = the number of standard deviations from the mean
   b = the slope of the regression line from the sample
   s_b = the standard error of the regression line
   n = the sample size
   df = n-2

4. **Calculate t***

   - 2nd Vars
   - Inverse t
   - Area = $\frac{(1-Confidence\ level)}{2}$
   - **df** = n-2

5. **Enter Data**

   - **Stat Edit**
   - Enter X into L_1
   - Enter Y into L_2

6. **Calculate Interval**

   - **Stat Tests**
   - LinRegTTest→Enter
   - X List- L_1
   - Y List- L_2
   - Freq 1
   - *C-level (confidence Level)*
   - *ReqEQ Alpha Trace*

7. **Write the interval**

8. **Write the Conclusion**

   We are _____% confident that the true population slope for _____
   Restate the definition of y
   relative to _____lies within the interval _____.
   Restate the definition of x

9. **Explain the meaning of the confidence level-***if asked*

   In repeated sampling, we expect that this method will capture the true population slope for
   _____ relative to _____ _____ percent of the time.
   Restate the definition of y *Restate the definition of x*

Notes: Confidence Interval for the Slope of the Regression Line (Data Scenario)

Scenario 1: Using the Data form the air fare and distance scenario create a 96% confidence interval for the slope of the regression line.

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Notes: Computer Print-outs for Regression

Source	Sum of Squares	df	Mean Square	F-ratio
Regression	26037.4	1	26037.4	14.9
Residual	27980	16	1748.79	

Variable	Coefficient	s.e. of Coeff	t-ratio	prob
Constant	177.215	19.99	8.86	≤0.0001
Distance	0.0786	0.0204	3.86	≤0.0014

R-squared = 48.2% R-squared (adjusted) = 45.0%

s = 41.82

1. What is the explanatory variable? What is the response variable?

   Identify the slope of the regression line

3. Identify the y-intercept

4. Write the Regression equation

5. Calculate the sample size.

6. Identify the standard deviation of the residuals. How could it be calculated using the values from the printout?

7. How can the t-statistic for the slope be calculated from the table values?

8. What is the p-value of the test? How can you verify with your calculator utilizing the information from the print-out?

9. What is the correlation coefficient?

Notes: Chi-Square Goodness of Fit

Facts About the Chi-Square Distribution

1. The curve is nonsymmetrical and skewed to the right.
2. There is a different chi-square curve for each degree freedom (this is like the t-distribution)
3. The test statistic for any test is always greater than or equal to zero.
4. When df > 90, the chi-square curve approximates the normal.
5. The mean, μ, is located just to the right of the peak.

Types of Chi-Square Tests:

1. **Chi-Square Goodness of Fit:** Tests to see if a sample distribution matches a known distribution.
   Key Words: *distribution, difference, historical, previous, model, matches*

2. **Chi-Square Test of Independence:** Tests to determine whether or not two or more variables from a single sample are associated/related/independent.
   Key words: *independence, association, relation*

3. **Chi-Square Test of Homogeneity:** Tests to see if a single variable sampled from 2 or more populations or groups has the ***same proportion.***
   Key words: ***proportions, ratios***

Chi Square Goodness of Fit Test

The Chi Square Goodness of Fit test is a hypothesis test used to determine if the data **"fit"** a particular distribution or not. **The null and the alternate hypotheses for this test may be written in sentences or may be stated as equations or inequalities. The goodness-of-fit test is almost always right tailed.**

The test statistic for a goodness-of-fit test is: $\chi^2 = \sum_{1}^{n} \frac{(O-E)^2}{E}$

where:

- O= observed values (data)
- E = expected values (from theory)
- n = the number of differen1. t data cells or categories
- Degrees of freedom are df = (number of categories - 1).

Chi-Square Goodness of Fit: Tests to see if a sample distribution matches some known distribution.

(χ^2 **GOF** has only one variable and one sample) **df = (number of categories - 1).**

H_0: ***The _____data fits/follows or matches a _____ distribution***

H_A: ***The _____ data does not fit/follow or match a _____ distribution***

Key words: ***distribution, difference, historical, previous, model, matches***

Recipe for Success: Chi-Square Goodness-of-Fit Hypothesis Test

1. Write your Hypothesis

- **Null** H_0: *The _____data follows a _____ distribution*
- **Alternative** H_A: *The _____ data does not follow a_____ distribution*
  Key words: *distribution, difference, historical, previous, model, matches*

2. Write the Conditions

1. Random Sample (1-sample with 1-variable)
2. n < 10% of the population
3. All expected values > 5
4. Counted data for observed

3. Write the Equation

$$\chi^2 = \frac{(O-E)^2}{E}$$

χ^2 = How far away a distribution is from the expected/Null
O = observed Value-the value from the sample
E = the expected or predicted value.
n = the number of categories

4. Calculate the Expected Values

- Sum the number observations in the sample
- Multiply the number of observations by the Historical **%**
  Note: *For the Expected value of a **Uniform Distribution**; Divide the sum of the sample observations by the number of categories*

5. List the Expected Values

6. Enter Data

- Stat Edit
- Enter Sample Data in column L_1
- Enter Expected Data in column L_2
- 2^{nd} Quit

7. Plug values into the equation
(1^{st} and last Expected values)

$$\chi^2 = \frac{(O-E)^2}{E} + \ldots + \frac{(O-E)^2}{E}$$

8. Calculate the χ^2 and the p-value

- Stat Tests
- χ^2 GOF-Test Enter
- Observed: Press 2^{nd} L_1
- Expected: Press 2^{nd} L_2
- **df= n-1** *(df is the degrees of freedom)*

9. State the Decision

- The p-value is_____
- If the p-value is less than alpha, Reject the Null
- If the p-value is greater than alpha, Fail to reject the Null

10. Write the Conclusion

Reject the Null: Our p-value is _____. We reject the Null. There is sufficient evidence at alpha = ___ to suggest that the is _____.
Restate H_A

Fail to Reject the Null: Our p-value is _____. We Fail to reject the Null. There is not sufficient evidence at alpha = ___ to suggest that is _____.
Restate H_A

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Notes: Chi-Square Goodness of Fit

Scenario 1: Absenteeism of college students from math classes is a major concern to math instructors because missing class appears to increase the drop rate. Three statistics instructors wondered whether the absentee rate was the **same** for every day of the school week. They took a random sample of absent students from three of their statistics classes during one week of the term. The results of the survey appear in the table.

Class Days	Monday	Tuesday	Wednesday	Thursday	Friday	
Number of absent students	28	22	18	20	32	

Run an appropriate hypothesis test to answer the professor's question.

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Recipe for Success: Chi-Square Goodness-of-Fit Hypothesis Test

1. Write your Hypothesis

- **Null** H_0: *The _____data follows a _____ distribution*
- **Alternative** H_A: *The _____ data does not follow a_____ distribution*
  Key words: *distribution, difference, historical, previous, model, matches*

2. Write the Conditions

1. Random Sample (1-sample with 1-variable)
2. n < 10% of the population
3. All expected values > 5
4. Counted data for observed

3. Write the Equation

$$\chi^2 = \frac{(O-E)^2}{E}$$

χ^2 = How far away a distribution is from the expected/Null
O = observed Value-the value from the sample
E = the expected or predicted value.
n = the number of categories

4. Calculate the Expected Values

- Sum the number observations in the sample
- Multiply the number of observations by the Historical **%**
  Note: *For the Expected value of a **Uniform Distribution**; Divide the sum of the sample observations by the number of categories*

5. List the Expected Values

6. Enter Data

- Stat Edit
- Enter Sample Data in column L_1
- Enter Expected Data in column L_2
- 2^{nd} Quit

7. Plug values into the equation
(1^{st} and last Expected values)

$$\chi^2 = \frac{(O-E)^2}{E} + \ldots + \frac{(O-E)^2}{E}$$

8. Calculate the χ^2 and the p-value

- Stat Tests
- χ^2 GOF-Test Enter
- Observed: Press 2^{nd} L_1
- Expected: Press 2^{nd} L_2
- **df= n-1** *(df is the degrees of freedom)*

9. State the Decision

- The p-value is_____
- If the p-value is less than alpha, Reject the Null
- If the p-value is greater than alpha, Fail to reject the Null

10. Write the Conclusion

Reject the Null: Our p-value is _____. We reject the Null. There is sufficient evidence at alpha = ___ to suggest that the is _____.
Restate H_A

Fail to Reject the Null: Our p-value is _____. We Fail to reject the Null. There is not sufficient evidence at alpha = ___ to suggest that is _____.
Restate H_A

Notes: Chi-Square Goodness of Fit

Scenario 2: A grocery store manager wishes to determine whether a certain product will sell equally well in any of five locations in the store. Five displays are set-up, one in each location, and the resulting numbers of the product sold are noted.

Location	**1**	**2**	**3**	**4**	**5**	
Number of Items Sold	**43**	**29**	**52**	**34**	**48**	

Is there evidence that location makes a difference? Test at the 10% level of significance.

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Recipe for Success: Chi-Square Goodness-of-Fit Hypothesis Test

1. Write your Hypothesis

- **Null** H_0: *The _____data follows a _____ distribution*
- **Alternative** H_A: *The _____ data does not follow a_____ distribution*
  Key words: *distribution, difference, historical, previous, model, matches*

2. Write the Conditions

1. Random Sample (1-sample with 1-variable)
2. n < 10% of the population
3. All expected values > 5
4. Counted data for observed

3. Write the Equation

$$\chi^2 = \frac{(O-E)^2}{E}$$

χ^2 = How far away a distribution is from the expected/Null
O = observed Value-the value from the sample
E = the expected or predicted value.
n = the number of categories

4. Calculate the Expected Values

- Sum the number observations in the sample
- Multiply the number of observations by the Historical **%**
  Note: *For the Expected value of a **Uniform Distribution**; Divide the sum of the sample observations by the number of categories*

5. List the Expected Values

6. Enter Data

- Stat Edit
- Enter Sample Data in column L_1
- Enter Expected Data in column L_2
- 2^{nd} Quit

7. Plug values into the equation
(1^{st} and last Expected values)

$$\chi^2 = \frac{(O-E)^2}{E} + \ldots + \frac{(O-E)^2}{E}$$

8. Calculate the χ^2 and the p-value

- Stat Tests
- χ^2 GOF-Test Enter
- Observed: Press 2^{nd} L_1
- Expected: Press 2^{nd} L_2
- **df= n-1** *(df is the degrees of freedom)*

9. State the Decision

- The p-value is_____
- If the p-value is less than alpha, Reject the Null
- If the p-value is greater than alpha, Fail to reject the Null

10. Write the Conclusion

Reject the Null: Our p-value is _____. We reject the Null. There is sufficient evidence at alpha = ___ to suggest that the is _____.
Restate H_A

Fail to Reject the Null: Our p-value is _____. We Fail to reject the Null. There is not sufficient evidence at alpha = ___ to suggest that is _____.
Restate H_A

Notes: Chi-Square Goodness of Fit

Scenario 3: In a recent year, at the 6pm time slot, television channels 2, 3, 4, and 5 captured the entire audience with 30%, 25%, 20%, and 25%, respectively. During the first week of the next session, 500 viewers are interviewed. The table records those observations.

Television Channels	2	3	4	5	Total
Number of viewers	139	138	112	111	500

Did the viewing preferences have a statistically significant change?

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Recipe for Success: Chi-Square Goodness-of-Fit Hypothesis Test

1. Write your Hypothesis

- **Null** H_0: *The _____data follows a _____ distribution*
- **Alternative** H_A: *The _____ data does not follow a_____ distribution*
  Key words: *distribution, difference, historical, previous, model, matches*

2. Write the Conditions

1. Random Sample (1-sample with 1-variable)
2. n < 10% of the population
3. All expected values > 5
4. Counted data for observed

3. Write the Equation

$$\chi^2 = \frac{(O-E)^2}{E}$$

χ^2 = How far away a distribution is from the expected/Null
O = observed Value-the value from the sample
E = the expected or predicted value.
n = the number of categories

4. Calculate the Expected Values

- Sum the number observations in the sample
- Multiply the number of observations by the Historical **%**
  Note: *For the Expected value of a **Uniform Distribution**; Divide the sum of the sample observations by the number of categories*

5. List the Expected Values

6. Enter Data

- Stat Edit
- Enter Sample Data in column L_1
- Enter Expected Data in column L_2
- 2^{nd} Quit

7. Plug values into the equation
(1^{st} and last Expected values)

$$\chi^2 = \frac{(O-E)^2}{E} + \ldots + \frac{(O-E)^2}{E}$$

8. Calculate the χ^2 and the p-value

- Stat Tests
- χ^2 GOF-Test Enter
- Observed: Press 2^{nd} L_1
- Expected: Press 2^{nd} L_2
- **df= n-1** *(df is the degrees of freedom)*

9. State the Decision

- The p-value is_____
- If the p-value is less than alpha, Reject the Null
- If the p-value is greater than alpha, Fail to reject the Null

10. Write the Conclusion

Reject the Null: Our p-value is _____. We reject the Null. There is sufficient evidence at alpha = ___ to suggest that the is _____.
Restate H_A

Fail to Reject the Null: Our p-value is _____. We Fail to reject the Null. There is not sufficient evidence at alpha = ___ to suggest that is _____.
Restate H_A

Notes: Chi-Square Goodness of Fit

Scenario 4: The number of televisions that American families have is distributed as follows:

Number of Televisions	Percent
0	10
1	16
2	55
3	11
4 or more	8

A random sample of 600 families in the far western United States resulted in the following data:

Number of Televisions	Frequency	
0	66	
1	119	
2	340	
3	60	
4 or more	15	

.

At the 1% significance level, does it appear that the distribution "number of televisions" of far western United States families is different from the distribution for the American population as a whole?

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Notes: Review of Independence

Independent Events-*Events that do not rely on one another. The outcome of one does not impact the outcome of the other.* ***If*** *independent* **P(A∩B) = P(A) × P(B)-Independent events are not associated, Independent events are not correlated and are not related.**

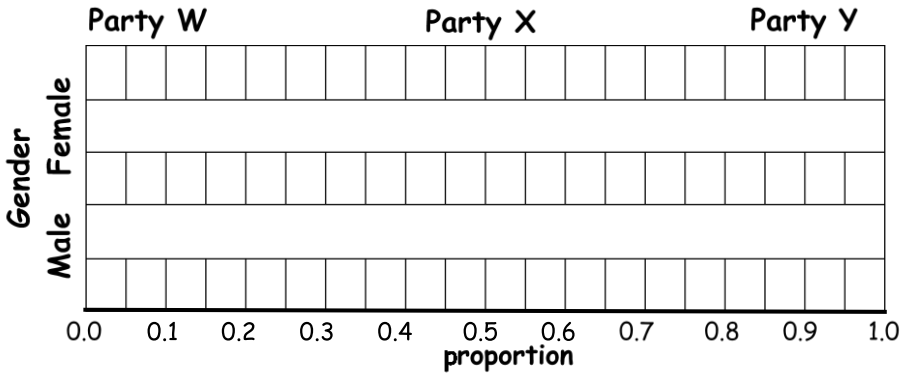
- ***Multiplication Principal-****if two events are independent their probabilities can be multiplied to find the likelihood that both events will occur at the same time.*
  Equation P(A∩B) = P(A) × P(B) this reads the probability of the intersection of A and B, often stated as *A and B,* is equal to the probability of A times the probability of B
- **Graphically:** The corresponding segments of a segment will be the same size because they have the same proportions.

Scenario: 2011 Problem 2 The table below shows the political party registration by gender of all 500 registered voters in Franklin Township.

PARTY REGISTRATION – FRANKLIN TOWNSHIP				
	Party W	Party X	Party Y	Total
Female	60	120	120	300
Male	28	124	48	200
Total	88	244	168	500

(b) Among the registered voters of Franklin Township, are the events "is a male" and "is registered for Party Y" independent?

c) In Lawrence Township, the proportions of all registered voters for Parties W, X, and Y are the same as for Franklin Township, and party registration is independent of gender. Complete the graph below to show the distributions of party registration by gender in Lawrence Township



Notes: Review of Independence

Scenario 2: Suppose A = a speeding violation in the last year and B = a car phone user. If A and B are independent then P (A AND B) = P (A)×P (B). A AND B is the event that a driver received a speeding violation last year and is also a car phone user. Suppose, in a study of drivers who received speeding violations in the last year and who use car phones, that 755 people were surveyed. Out of the 755, 70 had a speeding violation and 685 did not; 305 were car phone users and 450 were not.

If A and B are independent, then P (A AND B) = P(A) × P(B). Complete the table

	Car Phone Users	No Car Phone	Total
Speeding Ticket			70
No ticket			685
Total	305	450	755

About 28 people from the sample are expected to be car phone users and to receive speeding violations. In a test of independence, we state the null and alternate hypotheses in words. Since the contingency table consists of **two factors**, the null hypothesis states that the factors are **independent** and the alternate hypothesis states that they are **not independent (dependent)**. If we do a test of independence using the example above, then the null hypothesis is:

Ho: Being a car phone user and receiving a speeding violation are independent events.

Ha: Being a car phone user and receiving a speeding violation are not independent events or are associated.

If the null hypothesis were true, we would expect about 28 people to be car phone users and to receive a speeding violation.

Chi Square Test of Independence

The Chi Square Test of independence is a hypothesis test used to determine whether two factors are independent or not—whether or not an association exists. **The null and the alternate hypotheses for this test may be written in sentences. The test of independence is always right tailed.**

Chi-Square Test of Independence: Tests to see if 2 or more variables from a single sample are associated/related/independent. **df = (r – 1)(c – 1).** r=row & c=columns

H_0: *The*_____*and* _____*are independent*
Variables being compared

H_A: *The*_____*and* _____*are not independent*
Variables being compared

Key words: ***independence, association, relation***

The test statistic for a test of independence is: $\chi^2 = \sum_{1}^{n} \frac{(O-E)^2}{E}$

where:

- O= observed values (data)
- E = expected values (from theory) $E = \frac{\text{(row total)(column total)}}{\text{total number}}$
- n = the number of different data cells or categories
- **Degrees of freedom are df = (r – 1)(c – 1).**

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Recipe for Success: Chi-Square Test of Independence

1. **Write your Hypothesis**
   - **Null H_0:** *The_____and _____are independent*
     Variables being compared
   - **Alternative H_A:** *The_____and _____are associated and are not independent.*
     Variables being compared

   Key words: ***independence, association, relation***

2. **Write the Conditions**
   1. Random Sample (1-sample with 2-variable)
   2. n < 10% of the population
   3. All expected values > 5
   4. Counted data for observed

3. **Write the Equation**

   $$\chi^2 = \frac{(O-E)^2}{E}$$

   χ^2 = How far away a distribution is from the expected/Null
   O = observed Value-the value from the sample
   E = the expected or predicted value.
   n = the number of categories

4. **Calculate & List Expected Values**
   - 2nd Matrix→Edit →[A]
   - Enter Matrix size *(row x column) do not include totals.*
   - Enter observed Data
   - 2nd Quit
   - STAT TESTS
   - **C: χ^2-Test**
   - 2nd matrix→Edit ↓ [B] *(these are the expected values)*

5. **Plug values into the equation** (1st and last Expected values)

   $$\chi^2 = \frac{(O-E)^2}{E} + \ldots + \frac{(O-E)^2}{E}$$

6. **Calculate the χ^2 and the p-value**
   - STAT TESTS
   - **C: χ^2-Test**
   - **df= (r-1)(c-1)** *(r =rows & c=columns)*

7. **State the Decision**
   - The p-value is____
   - If the p-value is less than alpha, Reject the Null
   - If the p-value is greater than alpha, Fail to reject the Null

8. **Write the Conclusion**

Reject the Null: Our p-value is ____. We reject the Null. There is sufficient evidence at alpha = ___ to suggest that the _____are _____ are associated and are not independent.
Restate Variables

Fail to Reject the Null: Our p-value is ____. We Fail to reject the Null. There is not sufficient evidence at alpha = ___ to suggest that the _____ and _____ are not independent.
Restate Variables

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Notes: Chi-Square Test of Independence (Scenario)

Scenario 1: In a volunteer group, adults 21 and older volunteer from one to nine hours each week to spend time with a disabled senior citizen. The program recruits among community college students, four-year college students, and nonstudents. The following table is a random **sample** of the adult volunteers and the number of hours they volunteer per week.

Number of Hours Worked Per Week by Volunteer Type (Observed)

Type of Volunteer	1-3 Hours	4-6 Hours	7-9 Hours	Row Total
Community College Students	111	96	48	255
4-Year College Students	96	133	61	290
Nonstudents	91	150	53	294
Column Total	298	379	162	839

Are the number of hours volunteered **independent** of the type of volunteer?

Number of Hours Worked Per Week by Volunteer Type (Expected Values)

Type of Volunteer	1-3 Hours	4-6 Hours	7-9 Hours	Row Total
Community College Students				255
4-Year College Students				290
Nonstudents				294
Column Total	298	379	162	839

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Recipe for Success: Chi-Square Test of Independence

1. Write your Hypothesis

- **Null H_0:** *The_____and _____are independent*
  Variables being compared
- **Alternative H_A:** *The_____and _____are associated and are not independent.*
  Variables being compared

Key words: ***independence, association, relation***

2. Write the Conditions

1. Random Sample (1-sample with 2-variable)
2. n < 10% of the population
3. All expected values > 5
4. Counted data for observed

3. Write the Equation

$$\chi^2 = \frac{(O-E)^2}{E}$$

χ^2 = How far away a distribution is from the expected/Null
O = observed Value-the value from the sample
E = the expected or predicted value.
n = the number of categories

4. Calculate & List Expected Values

- 2nd Matrix→Edit →[A]
- Enter Matrix size *(row x column) do not include totals.*
- Enter observed Data
- 2nd Quit
- STAT TESTS
- **C: χ^2-Test**
- 2nd matrix→Edit ↓ [B] *(these are the expected values)*

5. Plug values into the equation
(1st and last Expected values)

$$\chi^2 = \frac{(O-E)^2}{E} + \ldots + \frac{(O-E)^2}{E}$$

6. Calculate the χ^2 and the p-value

- STAT TESTS
- **C: χ^2-Test**
- **df= (r-1)(c-1)** *(r =rows & c=columns)*

7. State the Decision

- The p-value is____
- If the p-value is less than alpha, Reject the Null
- If the p-value is greater than alpha, Fail to reject the Null

8. Write the Conclusion

Reject the Null: Our p-value is ____. We reject the Null. There is sufficient evidence at alpha = ___ to suggest that the _____are _____ are associated and are not independent.
Restate Variables

Fail to Reject the Null: Our p-value is ____. We Fail to reject the Null. There is not sufficient evidence at alpha = ___ to suggest that the _____ and _____ are not independent.
Restate Variables

Notes: Chi-Square Test of Independence (Scenario)

Scenario 2: To determine whether men with a combination of childhood abuse and a certain abnormal gene are more likely to commit violent crimes, a study is run on a simple random sample of 575 males in the 25 to 35 age group. The data are summarized in the following table:

	Not Abused, normal Gene	Abused, normal Gene	Not Abused, Abnormal Gene	Abused, Abnormal Gene	Totals
Criminal Behavior	48	21	32	26	
Normal Behavior	201	79	118	50	
Totals					

a. Is there evidence of a relationship between the four categories and behavior?
b. Is there evidence that among men with the normal gene, the proportion of abused men who commit violent crimes is greater than the proportion of non-abused men who commit violent crimes?
c. Is there evidence that among men who were not abused as children, the proportion of men with the abnormal gene who commit violent crimes is greater than the proportion of men with the normal gene who commit violent crimes?

Notes: Chi-Square Test of Independence (Scenario)

Scenario 3 2003 Question 5 A random sample of 200 students was selected from a large college in the United States. Each selected student was asked to give his or her opinion about the following statement.

"The most important quality of a person who aspires to be the President of the United States is a knowledge of foreign affairs."

Each response was recorded in one of five categories. The gender of each selected student was noted. The data are summarized in the table below:

	Response Category				
	Strongly Disagree	Somewhat Disagree	Neither Agree nor Disagree	Somewhat Agree	Strongly Agree
Male	10	15	15	25	25
Female	20	25	25	25	15

Is there sufficient evidence to indicate that the response is dependent on gender? Provide statistical evidence to support your conclusion.

Notes: Chi-Square Test of Homogeneity

Chi Square Test of Homogeneity

The Chi Square Test of Homogeneity is a hypothesis test used to compare samples from two or more populations with regard to a single categorical variable. The purpose of the test is to determine whether or not the proportion is the same for all populations being compared.

The null and the alternate hypotheses for this test may be written in sentences. The test of homogeneity is always right tailed.

Chi-Square Test of Homogeneity: Tests to see if a single variable sampled from 2 or more populations or groups has the ***same proportion***. **df = (r – 1)(c – 1).** r=row & c=columns

Expected values= (row total)(column total) ÷ (total number)

H_0: *The proportions are the same for*_____
The categories being compared

H_A: *The proportions are not the same for*_____
The categories being compared

Key words: ***proportions, ratios***

The test statistic for a test of independence is: $\chi^2 = \sum_{1}^{n} \frac{(O-E)^2}{E}$

where:

- O= observed values (data)
- E = expected values (from theory) $E = \frac{(\text{row total})(\text{column total})}{\text{total number}}$
- n = the number of different data cells or categories
- r = the number of rows in the table
- c = the number of columns in the table
- **Degrees of freedom are df = (r – 1)(c – 1).**

The **difference** between a **Chi-square test of independence** and a **Chi-square test of Homogeneity** is the design of the test.

- If the samples are from two or more populations and are comparing a single variable then the test is a Chi-square test of Homogeneity.
  Hint: Identify the populations and the number of samples

- If the test is drawn from a single sample and is comparing two or more variables to see if there is a relation then the test is a Chi-Square test of Independence.
  Hint: Identify the variables and make certain that there is only one sample

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Recipe for Success: Chi-Square Test of Homogeneity

1. Write your Hypothesis

- **Null H_0:** *The distribution of* _____ *and* _____ *are the same.*
  (1st sample; 2nd sample)
- **Alternative H_A:** *The distribution of* _____ *and* _____ *are not the same*
  (1st sample; 2nd sample)

Key words: ***distribution are the same; same***

2. Write the Conditions

1. Random Sample (2-sample with 1-variable)
2. n < 10% of the population
3. All expected values > 5
4. Counted data for observed

3. Write the Equation

$$\chi^2 = \frac{(O-E)^2}{E}$$

χ^2 = How far away a distribution is from the expected/Null
O = observed Value-the value from the sample
E = the expected or predicted value.
n = the number of categories

4. Calculate & List Expected Values

- 2nd Matrix→Edit →[A]
- Enter Matrix size *(row x column) do not include totals.*
- Enter observed Data
- 2nd Quit
- STAT TESTS
- **C: χ^2-Test**
- 2nd matrix→Edit ↓ [B] *(these are the expected values)*

5. Plug values into the equation
(1st and last Expected values)

$$\chi^2 = \frac{(O-E)^2}{E} + \ldots + \frac{(O-E)^2}{E}$$

6. Calculate the χ^2 and the p-value

- STAT TESTS
- **C: χ^2-Test**
- **df= (r-1)(c-1)** *(r =rows & c=columns)*

7. State the Decision

- The p-value is_____
- If the p-value is less than alpha, Reject the Null
- If the p-value is greater than alpha, Fail to reject the Null

8. Write the Conclusion

Reject the Null: Our p-value is _____. We reject the Null. There is sufficient evidence at alpha = ____ to suggest that the distributions of _____ (1st sample) and _____ (2nd sample) are the same.

Fail to Reject the Null: Our p-value is _____. We Fail to reject the Null. There is not sufficient evidence at alpha = ____ to suggest that the distributions of _____ (1st sample) and _____ (2nd sample) are not the same.

Notes: Chi-Square Test of Homogeneity

Scenario 1: In a large city a group of AP Statistics student work together on a project to determine which group of school employees has the greatest proportion who are satisfied with their jobs. In independent simple random samples of 100 teachers, 60 administrators, 45 custodians, and 55 secretaries, the numbers satisfied with their jobs were found to be 82, 38, 34, and 36 respectively. Is there evidence that employee job satisfaction varies by job category?

The observed counts are as follows:

	Satisfied	Not Satisfied	Totals
Teachers	82	18	
Administrators	38	22	
Custodians	34	11	
Secretaries	36	19	
Totals			

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Recipe for Success: Chi-Square Test of Homogeneity

1. **Write your Hypothesis**
   - **Null H_0:** *The distribution of* _____ *and* _____ *are the same.*
     (1st sample; 2nd sample)
   - **Alternative H_A:** *The distribution of* _____ *and* _____ *are not the same*
     (1st sample; 2nd sample)

   Key words: ***distribution are the same; same***

2. **Write the Conditions**
   1. Random Sample (2-sample with 1-variable)
   2. n < 10% of the population
   3. All expected values > 5
   4. Counted data for observed

3. **Write the Equation**

$$\chi^2 = \frac{(O-E)^2}{E}$$

χ^2 = How far away a distribution is from the expected/Null
O = observed Value-the value from the sample
E = the expected or predicted value.
n = the number of categories

4. **Calculate & List Expected Values**
   - 2nd Matrix→Edit →[A]
   - Enter Matrix size *(row x column) do not include totals.*
   - Enter observed Data
   - 2nd Quit
   - STAT TESTS
   - **C: χ^2-Test**
   - 2nd matrix→Edit ↓ [B] *(these are the expected values)*

5. **Plug values into the equation**
   (1st and last Expected values)

$$\chi^2 = \frac{(O-E)^2}{E} + \ldots + \frac{(O-E)^2}{E}$$

6. **Calculate the χ^2 and the p-value**
   - STAT TESTS
   - **C: χ^2-Test**
   - **df= (r-1)(c-1)** *(r =rows & c=columns)*

7. **State the Decision**
   - The p-value is_____
   - If the p-value is less than alpha, Reject the Null
   - If the p-value is greater than alpha, Fail to reject the Null

8. **Write the Conclusion**

Reject the Null: Our p-value is _____. We reject the Null. There is sufficient evidence at alpha = ____ to suggest that the distributions of _____ (1st sample) and _____ (2nd sample) are the same.

Fail to Reject the Null: Our p-value is _____. We Fail to reject the Null. There is not sufficient evidence at alpha = ____ to suggest that the distributions of _____ (1st sample) and _____ (2nd sample) are not the same.

Notes: Chi-Square Test of Homogeneity

Scenario 2: Surveys of drivers of differing age groups were taken to determine if the driver had been in an accident during the previous year, and if so was it a minor or major accident. The results are tabulated below.

		Accident Type		
AGE	None	minor	major	**Totals**
under 18	67	10	5	
18-25	42	6	5	
26-40	75	8	4	
40-65	56	4	6	
over 65	57	15	1	
Totals				

Is there evidence that the accident type varies by age?

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Recipe for Success: Chi-Square Test of Homogeneity

1. Write your Hypothesis

- **Null H_0:** *The distribution of* _____ *and* _____ *are the same.*
  (1st sample; 2nd sample)
- **Alternative H_A:** *The distribution of* _____ *and* _____ *are not the same*
  (1st sample; 2nd sample)

Key words: ***distribution are the same; same***

2. Write the Conditions

1. Random Sample (2-sample with 1-variable)
2. n < 10% of the population
3. All expected values > 5
4. Counted data for observed

3. Write the Equation

$$\chi^2 = \frac{(O-E)^2}{E}$$

χ^2 = How far away a distribution is from the expected/Null
O = observed Value-the value from the sample
E = the expected or predicted value.
n = the number of categories

4. Calculate & List Expected Values

- 2nd Matrix→Edit →[A]
- Enter Matrix size *(row x column) do not include totals.*
- Enter observed Data
- 2nd Quit
- STAT TESTS
- **C: χ^2-Test**
- 2nd matrix→Edit ↓ [B] *(these are the expected values)*

5. Plug values into the equation
(1st and last Expected values)

$$\chi^2 = \frac{(O-E)^2}{E} + \ldots + \frac{(O-E)^2}{E}$$

6. Calculate the χ^2 and the p-value

- STAT TESTS
- **C: χ^2-Test**
- **df= (r-1)(c-1)** *(r =rows & c=columns)*

7. State the Decision

- The p-value is_____
- If the p-value is less than alpha, Reject the Null
- If the p-value is greater than alpha, Fail to reject the Null

8. Write the Conclusion

Reject the Null: Our p-value is _____. We reject the Null. There is sufficient evidence at alpha = ____ to suggest that the distributions of _____ (1st sample) and _____ (2nd sample) are the same.

Fail to Reject the Null: Our p-value is _____. We Fail to reject the Null. There is not sufficient evidence at alpha = ____ to suggest that the distributions of _____ (1st sample) and _____ (2nd sample) are not the same.

Notes: Chi-Square Test of Homogeneity

Scenario 3: One random sample indicates that the number of televisions that American families have is distributed (this is the **given** distribution for the American population) as follows:

Number of Televisions	Percent	Frequency
0	10	60
1	16	96
2	55	330
3	11	66
4 or more	8	48

A second random sample in the far western United States resulted in the following data:

Number of Televisions	Frequency
0	66
1	119
2	340
3	60
4 or more	15

At the 1% significance level, does it appear that the distributions of televisions are the same in the far western United States and the American population as a whole?

Recipe for Success: 1 Sample T-Distribution (Confidence Interval)

1. **Define the parameter μ *in context***
2. **Write the Conditions**
   - **Random Sample**
   - **n < 10% of the population**
   - **Normal Population or $n \geq 30$**

   If data is given, draw a boxplot or histogram-to show normality
3. **Write the Equation**

   $$\bar{x} \pm t^* \left(\frac{s}{\sqrt{n}}\right)$$

   t = the number of standard deviations a value is from the mean
   $\bar{x}$ = the mean of the sample.
   s = the standard deviation of the sample
   n = the size of the sample
4. **Graph and Shade**
5. **Enter the Data if Given**
   - Stat Edit
   - Enter Data in column L1
   - 2nd Quit
   - Stat Calc
   - 1-Var Stats: $\bar{x}$, s_x, **n**
6. **Identify and label all inputs**
   - s comes from the problem or the data
   - $\bar{x}$ comes from the problem or the data
   - **n** comes from the problem or the data
   - **df** = n-1
7. **Calculate *t**
   - 2nd Vars
   - Inverse t
   - Area = $\frac{(1-Confidence\ level)}{2}$
   - **df** = n-1
8. **Plug in the values**
9. **Calculate the Interval**
   - Stat Tests
   - **8:T Interval**
   - Highlight **Stats** *(Highlight Data if data is given)*
   - $\bar{x}$ comes from the problem or the data
   - s_x come from the problem or the data
   - **n** comes from the problem or the data
   - Inter the confidence level
10. **Write the Interval**
11. **Write the Conclusion**

    We are _____% confident that the true population mean for _____ lies within the interval _____. *Restate the definition of the mean*
12. **Explain the meaning of the confidence level-***if asked*

    In repeated sampling, we expect that this method will capture the true population mean _____percent of the time.
    Restate the Confidence Level

Recipe for Success: 1 Sample T-Distribution (Hypothesis Test)

1. **Write your Hypothesis**
   - Null H_0: $\mu =$
   - Alternative H_A: $\mu \neq$ or $<$ or $>$
2. **Define the parameter μ *in context***
3. **Write the Conditions**
   1. **Random Sample**
   2. **n < 10% of the population**
   3. **Normal Population or *n > 30***

   If data is given, draw a boxplot or histogram-to show normality
4. **Write the Equation**

$$t = \frac{\overline{x} - \mu}{\frac{s}{\sqrt{n}}}$$

t = the number of standard deviations a value is from the mean
μ = the mean of the population or what is assumed to be true
$\overline{x}$ = the mean of the sample.
s = the standard deviation of the sample
n = the size of the sample

5. **Draw the graph and Shade**
6. **Enter Data** (if given)
   - Stat Edit
   - Enter Data in column L1
   - 2nd Quit
   - Stat Calc
   - 1-Var Stats: $\overline{x}$, s_x, **n**
7. **List & Label all of input values**
8. **Plug values into the equation**
9. **Calculate the t and the p-value**

   df= n-1

   (df is the degrees of freedom)
   - Stat Tests
   - **2:T-Test Enter**
   - Highlight **Stats** *(Highlight Data if data is given)*
   - μ comes from the problem
   - $\overline{x}$ comes from the problem or the data
   - s_x come from the problem or the data
   - **n** comes from the problem or the data
   - Choose $\neq$ or $<$ or $>$ (use Shaded graph of H_A)
10. **State the Decision**
    - The p-value is____
    - If the p-value is less than alpha, Reject the Null
    - If the p-value is greater than alpha, Fail to reject the Null
11. **Write the Conclusion**

Reject the Null: Our p-value is ____. We reject the Null. There is sufficient evidence at alpha = ___ to suggest that the true population mean for _____is _____

Restate the definition of the mean *Restate H_A $\neq$ or $<$ or $>$ mean*

Fail to Reject the Null: Our p-value is ____. We Fail to reject the Null. There is not sufficient evidence at alpha = ___ to suggest that the true population mean for

_____is _____

Restate the definition of the mean *Restate H_A $\neq$ or $<$ or $>$ mean*

Recipe for Success: Hypothesis Test for a Paired T-test

1. **Write your Hypothesis**
   - Null H_0: $\mu_d = 0$
   - Alternative H_A: $\mu_d \neq$ or $<$ or > 0

2. **Define parameter μ_d *in context* & write the conditions**
   1. **Random Sample**
   2. **n < 10% of the population**
   3. **Differences are independent**
   4. **The differences are Normal or *n > 30***

   If data is given, draw a boxplot or histogram-to show normality

3. **Write the Equation**

$$t = \frac{\bar{x}_d - \mu_d}{\frac{s_d}{\sqrt{n}}}$$

   t = the number of standard deviations a value is from the mean
   μ_d = 0
   $\bar{x}_d$ = the mean of the sample differences
   s_d = the standard deviation of the sample differences
   n = the number of sample differences

4. **Enter Data** (if given)
   - **Stat Edit**
   - **Enter Data** in columns L_1 & L_2

5. **Find $\bar{x}_d$**
   - At the top of L_3 type 2^{nd} L_1 - 2^{nd} L_2

6. **List & Label all of input values**
   n, $\bar{x}_d$, s_d
   μ_d = 0 & **df= n-1**
   - **Stat Calc**
   - **1-Var Stats** press **Enter**
   - **List** type 2^{nd} L_3

7. **Plug values into the equation**

8. **Calculate the t and the p-value**
   μ_d = 0
   df= n-1
   - Stat Tests
   - **2**:T-Test Enter
   - Highlight **Data** if data is used otherwise highlight **STATS**
   - s_d come from the problem or the data
   - $\bar{x}_d$ comes from the problem or the data
   - **n** comes from the problem or the data
   - Choose $\neq$ or $<$ or $>$ (look for key words)

9. **State the Decision**
   - The p-value is_____
   - If the p-value is less than alpha, Reject the Null
   - If the p-value is greater than alpha, Fail to reject the Null

10. **Write the Conclusion**

Reject the Null: Our p-value is _____. We reject the Null. There is sufficient evidence at alpha = _____ to suggest that the true population mean differences for

_____is _____

Restate the definition of the mean differences *Restate H_A $\neq$ or $<$ or $>$ mean differences*

Fail to Reject the Null: Our p-value is _____. We Fail to reject the Null. There is not sufficient evidence at alpha = ___ to suggest that the true population mean difference for

_____is _____

Restate the definition of the mean difference *Restate H_A $\neq$ or $<$ or $>$ mean difference*

Recipe for Success: Confidence Interval for a Paired T-test

1. **Define parameter μ_d *in context***

2. **Write the Conditions**
   1. **Random Sample**
   2. **n < 10% of the population**
   3. **Differences are independent**
   4. **The differences are Normal or *n > 30***

   If data is given, draw a boxplot/ histogram-to show normality

3. **Write the formula for the Test**

   $$\bar{x}_d \pm t^* \frac{s_d}{\sqrt{n}}$$

   t* = the number of standard deviations a value is from the mean & is based on the Confidence Level

   μ_d = 0

   $\bar{x}_d$ = the mean of the sample differences

   s_d = the standard deviation of the sample differences

   n = the number of sample differences

4. **Enter the Data if Given**
   - **Stat Edit**
   - **Enter Data** in columns **L_1 & L_2**

5. **Find $\bar{x}_d$**
   - **Stat Edit**
   - At the top of **L_3** type **2^{nd} L_1 - 2^{nd} L_2**

6. **Identify & label all inputs.**
   - s_d come from the problem or the data
   - $\bar{x}_d$ comes from the problem or the data
   - **n** comes from the problem or the data
   - **df= n-1** *(df is the degrees of freedom)*

7. **Calculate t***
   - 2^{nd} Vars
   - Inverse t
   - Area = $\frac{(1-Confidence\ level)}{2}$
   - **df** = n-1

8. **Plug in and calculate the Confidence Interval**
   - **STAT**
   - **TESTS**
   - 8: **T Interval**

9. **Write the Interval**

10. **Write the Conclusion**

We are _____% confident that the true population mean difference for _____ lies within the interval _____.

Restate the definition of the mean differences

Recipe for Success: Type I Errors

Type I Error-The probability of Rejecting the null given that the null is True.

1. **Write the Hypothesis**
   - Null H_0:
   - Alternative H_A:

α = the probability of committing a Type I error
β = the probability of committing a Type II error
1- β = the power of the test

2. **Define parameter (μ or p) in context**

3. **Define a Type I Error α**

Definition: *The probability of rejecting the Null given that the Null is true.*

Remember: To commit a **Type I Error**, we must have **rejected the null** and were incorrect

simplified definition: *Rejecting a true Null and Accepting a False Alternative*

4. **Explain a Type I Error in Context of the problem**

In this case, the probability of rejecting_____(*Restate* H_0)in favor of _____(*Restate* H_A)given the fact_____(*Restate* H_0)is true.

5. **Explain the consequences of Committing a Type I Error**

The consequences for committing at Type I Error are…
Explanation must be in the context and in simplified language.

Methods of Decreasing Type I Errors-α

1. **Decrease α – the level of significance**
   - Increases β-the probability of a Type II Error
   - More likely to accept a false null-*(this is an error)*
   - Power Decreases
2. **Decrease Power**
   - More likely to accept a false null—(Type II increases: negative)
   - Less likely to reject a true null—(Type I decreases: positive)
3. **Increase Sample Size**
   - Decreases Type I Error
   - Decreases Type II Error
   - Increases Power
   - (Costs money and Time)

In General:

As α ↓, power ↓, & β ↑

And

As α ↑, power ↑, & β ↓

P-value

1. **Write the Hypothesis**
   - Null H_0:
   - Alternative H_A:

2. **Define parameter (μ or p) in context**

3. **Define P-value**

P-value is the probability of getting a test statistic as extreme or more extreme given that the null is true.

4. **Explain P-value in the Context of the problem**

There is a _____(*P-value*)% chance that we would get a test statistic this extreme in favor of_____(*Restate* H_A) when in fact_____(*Restate* H_0) is true

Recipe for Success: Type II Errors

Type II Error-The probability of Accepting the null given that the null is False.

1. **Write the Hypothesis**
   - Null H_0:
   - Alternative H_A:

α = the probability of committing a Type I error
β = the probability of committing a Type II error
1- β = the power of the test

2. **Define parameter (μ or p) in context**

3. **Define a Type II Error β**

Remember: to commit a **Type II Error** we **failed to reject** the null and were incorrect.

Definition: The probability of accepting the Null given that it is false.

simplified definition: Accepting a False Null and Rejecting a True Alternative

4. **Explain Type II Error in Context of the problem**

In this case, the probability of accepting_____given that
Restate H_0
_____is false and _____is true.
Restate H_0 *Restate* H_A

5. **Explain the consequences of Committing a Type II Error**

The consequences for committing a Type II Error are…

Explanation must be in the context and in simplified language.

Methods of Decreasing Type II Errors- β

As β ↓, power ↑, & α ↑

And

As β ↑, power ↓, & α ↓

1. **Increase α – the level of significance**
   - Increases-the probability of a Type I Error α
   - More likely to reject a True Null—*(this is an error)*
   - Power Increases
2. **Increase Power**
   - More likely to reject a true null—(Type I increases: negative)
   - Less likely to accept a false null—(Type II decreases: positive)
3. **Increase Sample Size**
   - Decreases Type I Error
   - Decreases Type II Error
   - Increases Power
   - (Costs money and Time)

P-value

1. **Write the Hypothesis**
   - Null H_0:
   - Alternative H_A:
2. **Define parameter (μ or p) in context**
3. **Define P-value**

P-value is the probability of getting a test statistic as extreme or more extreme given that the null is true.

4. **Explain P-value in the Context of the problem**

There is a _____% chance that we would get a test statistic
P-value
this extreme in favor of_____ when in fact_____is true.
Restate H_A *Restate* H_0

Recipe for Success: 2-Sample T Hypothesis Test (difference of Means)

1. **Write the Hypothesis**
   - Null H_0: $\mu_1 = \mu_2$
   - Alternative H_A: $\mu_1 \neq \mu_2$ or $\mu_1 < \mu_2$ or $\mu_1 > \mu_2$
2. **Define μ_1 & μ_2** *in context*
3. **Write the Conditions**
   1. **Both samples are random**
   2. **n < 10% of the population**
   3. **Populations are independent**
   4. **Normal Population or *n > 30***

   If data is given, draw a boxplot or histogram-to show normality
4. **Write the Equation**

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

t = the number of standard deviations from the mean
μ_1 & μ_2 = the means of the population (may be assumed)
$\bar{x}_1$ & $\bar{x}_2$ = the means of the samples
s_1 & s_2 = the standard deviation of the sample
n_1 & n_2 = the size of the sample

5. **Enter Data** (if given)
   - Stat Edit
   - Enter Data in columns L_1 & L_2
   - 2nd Quit
   - Stat Calc
   - 1-Var Stats L_1: $\bar{x}_1$, s_1, n_1 and L_2: $\bar{x}_2$, s_2, n_2
6. **List & Label all of input values**

   $\bar{x}_1$, s_1, n_1, $\bar{x}_2$, s_2, n_2

   df (comes from the calculator)
   - Stat Tests
   - **4**: 2-SampTTest Enter
   - Highlight **Data** if data is used otherwise highlight **STATS**
   - **s_1 & s_2** comes from the problem or the data
   - **$\bar{x}_1$ & $\bar{x}_2$** comes from the problem or the data
   - **n_1 & n_2** comes from the problem or the data
   - **pooled** highlight no
   - **Choose ≠ or < or >** (look for key words)
7. **Plug values into the equation**
8. **Write the t and the p-value** The T and the p-value are calculated in step 5
9. **State the Decision**
   - The p-value is_____
   - compare to alpha: p-value (< or >) **alpha**
   - If the p-value is **less** than alpha, **Reject the Null**
   - If the p-value is **greater** than alpha, **Fail to reject the Null**
10. **Write the Conclusion**

Reject the Null: Our p-value is ____. We reject the Null. There is sufficient evidence at alpha = ___ to suggest that the difference in the true population mean for _____ is _____
Restate the definition of the 1st mean *Restate H_A ≠ or < or > 2nd mean definition*

Fail to Reject the Null: Our p-value is ____. We Fail to reject the Null. There is not sufficient evidence at alpha = ____ to suggest that the difference in true population mean for _____is _____
Restate the definition of the 1st mean *Restate H_A ≠ or < or > 2nd mean definition*

Recipe for Success: 2 Sample T-Confidence Intervals

1. **Define μ_1 & μ_2** *in context*
2. **Write the Conditions**

   1. **Both samples are random**
   2. **n < 10% of the population**
   3. **Populations are independent**
   4. **Normal Population or *n > 30***

   If data is given, draw a boxplot or histogram-to show normality

3. **Write the formula for the Test**

$$\bar{x}_1 - \bar{x}_2 \ \pm \ t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$\bar{x}_1$ **&** $\bar{x}_2$ = the means of the samples
s1 & s2 = the standard deviation of the sample
n1 & n2 = the size of the sample

4. **Graph and Shade**
5. **Enter the Data if Given**

- Stat Edit
- Enter Data in columns L_1 & L_2
- 2nd Quit
- Stat Calc
- 1-Var Stats L_1: $\bar{x}_1$, s_1, n_1 and L_2: $\bar{x}_2$, s_2, n_2

6. **List & Label all of input values**
   $\bar{x}_1$, **s1, n1,** $\bar{x}_2$, **s2, n2, df**

   df (comes from the calculator)

- Stat Tests
- **0**:2-Samp T Int
- Highlight **Data** if data is used otherwise highlight **STATS**
- **s1 & s2** comes from the problem or the data
- $\bar{x}_1$ **&** $\bar{x}_2$ comes from the problem or the data
- **n1 & n2** comes from the problem or the data
- **pooled** highlight no

7. **Calculate *t**

- 2nd Vars
- Inverse t
- Area = $\frac{(1-Confidence\ level)}{2}$
- **df** (comes from the calculator in the step above)

8. **Plug in the values**
9. **Write the interval**
10. **Write the Conclusion**

We are _____% confident that the true population mean difference for_____
Restate the definition of the μ_1
and _____ lies within the interval _____
Restate the definition of the μ_2

11. **Explain the meaning of the confidence level-***if asked*

In repeated sampling we expect this method to capture the true population mean difference for _____ and _____ _____% of the time.
Restate the definition of the μ_1 *Restate the definition of the* μ_2

Recipe for Success: 1 Sample Proportion Hypothesis Test

1. **Write the Hypothesis**
   - Null H_0: p_0 =
   - Alternative H_A: $p_0 \neq$ or $<$ or $>$
2. **Define parameter p_0 *in context***
3. **Write the Conditions**
   - **Simple Random Sample**
   - **$n\hat{p} \geq 10$**
   - **$n\hat{q} \geq 10$**
   - **n is less than 10% of the population $\frac{n}{.1}$**
4. **Write the Equation**

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{(p_0)(q_0)}{n}}}$$

z = the number of standard deviations a value is from the center
p_0 = the population **proportion** or what is assumed to be true
q_0 = **1 - p_0** (q_0 = the expected proportion of failures)
n = the size of the sample
x = the number of successes

5. **Draw the graph and Shade H_A**

$\hat{p} = \frac{x}{n}$ sample proportion of successes-those that met criteria
$\hat{q}$ = **1 - $\hat{p}$** the sample proportion of failures

6. **List & Label all of input values**
   - p_0 should be given
   - q_0 = **1 - p_0**
   - x = the number of successes from the sample
   - n = the sample size
   - $\hat{p} = \frac{x}{n}$
   - $\hat{q}$ = **1 - $\hat{p}$**
7. **Plug values into the equation**
8. **Calculate the z and the p-value**
   - Stat Tests
   - 1-proportion z-test
   - p_0 comes from the problem
   - x comes from the problem or the data
   - **n** comes from the problem or the data
   - Choose $\neq$ or $<$ or $>$ *(using Shaded graph of H_A)*
9. **State the Decision**
   - The p-value is____
   - If the p-value is less than alpha, Reject the Null
   - If the p-value is greater than alpha, Fail to reject the Null
10. **Write the Conclusion**

Reject the Null: Our p-value is ____. We reject the Null. There is sufficient evidence at alpha = ___ to suggest that the true population proportion for _____
Restate the definition of the p_0
is _____
Restate H_A $\neq$ or $<$ or $>$ p_0

Fail to Reject the Null: Our p-value is ____. We Fail to reject the Null. There is not sufficient evidence at alpha = ___ to suggest that the true population mean for
_____is _____
Restate the definition of the p_0 *Restate H_A $\neq$ or $<$ or $>$ p_0*

Recipe for Success: 1 Sample Proportions (Confidence Interval)

1. **Define parameter p_0** *(the population proportion)* **in context**

2. **Write the Conditions**
   - **Simple Random Sample**
   - **$n\hat{p} \geq 10$**
   - **$n\hat{q} \geq 10$**
   - **n is less than 10% of the population $\frac{n}{.1}$**

3. **Write the Equation**

$$\hat{p} \pm z^* \sqrt{\frac{(\hat{p})(\hat{q})}{n}}$$

4. **List the Values**

   $\hat{p} = \frac{x}{n}$ sample proportion of successes-those that met criteria
   $\hat{q} = 1 - \hat{p}$ the sample proportion of failures
   z* = the number of standard deviations a value is from the center
   x = the number of successes or measured outcomes of interest
   n = the size of the sample

5. **Calculate z***
   - 2nd Vars
   - Inverse Norm
   - Area = $\frac{(1-Confidence\ level)}{2}$
   - $\mu = 0$ and $\sigma = 1$

6. **Plug in the values**

7. **Calculate the Interval**
   - Stat Tests
   - **1-PropZInt**
   - ***x*** comes from the problem or the data
   - **n** comes from the problem or the data
   - **C-Level** Confidence level comes from the problem

8. **Write the interval**

9. **Write the Conclusion**

   We are _____% confident that the true population proportion for _____ lies within the interval _____.

   Restate the definition of the p_0

10. **Explain the meaning of the confidence level-***if asked*

    In repeated sampling, we expect that this method will capture the true population proportion _____percent of the time.

    Restate the Confidence Level

Recipe for Success: 1 Sample Proportion Hypothesis Test

1. **Write the Hypothesis**
   - Null H_0: p_0 =
   - Alternative H_A: $p_0 \neq$ or $<$ or $>$
2. **Define parameter p_0 *in context***
3. **Write the Conditions**
   - **Simple Random Sample**
   - **$n\hat{p} \geq 10$**
   - **$n\hat{q} \geq 10$**
   - **n is less than 10% of the population $\frac{n}{.1}$**
4. **Write the Equation**

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{(p_0)(q_0)}{n}}}$$

z = the number of standard deviations a value is from the center
p_0 = the population **proportion** or what is assumed to be true
q_0 = **1 - p_0** (q_0 = the expected proportion of failures)
n = the size of the sample
x = the number of successes

5. **Draw the graph and Shade H_A**

$\hat{p} = \frac{x}{n}$ sample proportion of successes-those that met criteria
$\hat{q}$ = **1 - $\hat{p}$** the sample proportion of failures

6. **List & Label all of input values**
   - p_0 should be given
   - q_0 = **1 - p_0**
   - x = the number of successes from the sample
   - n = the sample size
   - $\hat{p} = \frac{x}{n}$
   - $\hat{q}$ = **1 - $\hat{p}$**
7. **Plug values into the equation**
8. **Calculate the z and the p-value**
   - Stat Tests
   - 1-proportion z-test
   - p_0 comes from the problem
   - x comes from the problem or the data
   - **n** comes from the problem or the data
   - Choose $\neq$ or $<$ or $>$ *(using Shaded graph of H_A)*
9. **State the Decision**
   - The p-value is_____
   - If the p-value is less than alpha, Reject the Null
   - If the p-value is greater than alpha, Fail to reject the Null
10. **Write the Conclusion**

Reject the Null: Our p-value is _____. We reject the Null. There is sufficient evidence at alpha = ___ to suggest that the true population proportion for _____
Restate the definition of the p_0
is _____
Restate H_A $\neq$ or $<$ or $>$ p_0

Fail to Reject the Null: Our p-value is _____. We Fail to reject the Null. There is not sufficient evidence at alpha = ___ to suggest that the true population mean for
_____is _____
Restate the definition of the p_0 *Restate H_A $\neq$ or $<$ or $>$ p_0*

Recipe for Success: Confidence Interval-2 Sample Proportions

1. Define p_1 & p_2 in context *(the population proportions)*

p_1 = the population **proportion** for the 1st proportion
P_2 = the population **proportion** for the 2nd proportion

2. Write the Conditions *(must be for both sets of data)*

- **Independent Random Samples**
- **$n\hat{p} \geq 10$**
- **$n\hat{q} \geq 10$**
- **n is less than 10% of the population $\frac{n}{.1}$**

3. Write the Equation

$$\hat{p}_1 - \hat{p}_2 \pm z^* \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

4. List the Values

z = the number of standard deviations a value is from the center
n_1 = the size of the sample of the 1st proportion
x_1 = the number of outcomes of interest of the 1st proportion
$\hat{p}_1 = \frac{x_1}{n_1}$ 1st sample proportion of interest
n_2 = the size of the sample of the 2nd proportion
x_2 = the number of outcomes of interest of the 2nd proportion
$\hat{p}_2 = \frac{x_2}{n_2}$ 2nd sample proportion of interest

5. Calculate z*

- 2nd Vars
- Inverse Norm
- Area = $\frac{(1-Confidence\ level)}{2}$
- $\mu = 0$ and $\sigma = 1$

6. Plug in the values

7. Calculate the Interval

- Stat Tests
- **2-PropZInt**
- x_1 comes from the problem or the data
- n_1 comes from the problem or the data
- x_2 comes from the problem or the data
- n_2 comes from the problem or the data
- **C-Level** Confidence level comes from the problem

8. Write the interval

9. Write the Conclusion

We are _____% confident that the true population proportion difference between _____ and _____ lies within the interval _____.

Restate the definition of the p_1 *Restate the definition of the p_2*

10. Determining significance.

- **If 0 is in the interval**—There is **No** significant difference
- **If 0 is not in the interval**-There **Is** a significant difference

Recipe for Success: Hypothesis Test: 2 Sample Proportions

1. **Write your Hypothesis**
   - Null H_0: $p_1 = p_2$
   - Alternative H_A: $p_1 \neq$ or $<$ or $>$ p_2
2. **Define p_1 & p_2 in context**
   - p_1 = the population **proportion** for the 1st proportion
   - P_2 = the population **proportion** for the 2nd proportion
3. **Write the Conditions** *(must be for both sets of data)*
   - **Independent Random Samples**
   - **n is less than 10% of the population** $\frac{n}{.1}$
   - $n\hat{p} \geq 10$
   - $n\hat{q} \geq 10$
4. **Write the Equations**

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{(\hat{p}_c)(\hat{q}_c)}{n_1} + \frac{(\hat{p}_c)(\hat{q}_c)}{n_2}}}$$

$$\hat{p}_c = \frac{x_1 + x_2}{n_1 + n_2}$$

z = the number of standard deviations a value is from the center
n_1 = the size of the sample of the 1st proportion
x_1 = the number of outcomes of interest of the 1st proportion
$\hat{p}_1 = \frac{x_1}{n_1}$ 1st sample proportion of interest
n_2 = the size of the sample of the 2nd proportion
x_2 = the number of outcomes of interest of the 2nd proportion
$\hat{p}_2 = \frac{x_2}{n_2}$ 2nd sample proportion of interest
$\hat{p}_c$ = the 2 combined or pooled proportion successes
$\hat{q}_c = 1 - \hat{p}_c$ the 2 combined or pooled proportion successes

5. **List & Label all of input values**
   Calculate $\hat{p}_1$ & $\hat{p}_2$ & $\hat{p}_c$ & $\hat{q}_c$
6. **Plug values into the equation**
7. **Calculate the z and the p-value**
   - Stat Tests
   - 2-proportion z-test
   - x_1 & x_2 comes from the problem or the data
   - n_1 & n_2 comes from the problem or the data
   - Choose $\neq$ or $<$ or $>$
8. **State the Decision**
   - The p-value is_____
   - If the p-value is less than alpha, Reject the Null
   - If the p-value is greater than alpha, Fail to reject the Null
9. **Write the Conclusion**

Reject the Null: Our p-value is _____. We reject the Null. There is sufficient evidence at alpha = _____ to suggest that the true population proportion _____ is
Restate the definition of the p_1
_____ **true population proportion** _____.
Restate H_A $\neq$ or $<$ or $>$ *Restate the definition of the p_2*

Fail to Reject the Null: Our p-value is _____. We Fail to reject the Null. There is not sufficient evidence at alpha = _____ to suggest that the true population proportion for
_____is _____ than the true population proportion for
Restate the definition of the p_1 *Restate H_A $\neq$ or $<$ or $>$*
_____.
Restate the definition of the p_2

Recipe for Success: Definition Template for Regression

Slope β_1 = m = $\frac{\Delta y}{\Delta x}$ (using y = m x + b notation)

_____ is the expected amount of change in _____
Value of the slope *Restate the definition of Y*

for a 1 unit change in _____ .
Restate the definition of x

Slope Intercept β_0 = b = constant (using y = m x + b notation)

The amount of _____ that you begin with or would have if
Restate the definition of Y
the amount of_____ = zero.
Restate the definition of x

Correlation Coefficient (r) = $\sqrt{R^2}$

- **$|r| \geq .75$** There is a strong _____linear relationship between
(+ or –)--*use the sign of the slope*
_____ and _____.
Restate the definition of Y *Restate the definition of x*

$.40 < |r| < .75$ There is a moderately strong _____linear
(+ or –)--*use the sign of the slope*
relationship between _____and_____.
Restate the definition of Y *Restate the definition of x*

- **$|r| < .40$** There is a weak _____linear relationship between
(+ or –)--*use the sign of the slope*
_____ and _____.
Restate the definition of Y *Restate the definition of x*

Coefficient of Determination (R^2)

_____% of the variation in the _____ can be explained by
Restate the definition of Y
changes in the_____
Restate the definition of x

S The **standard deviation of the residuals** is _____ and measures the variance in
_____ for a given amount of _____.
Restate the definition of Y *Restate the definition of x*

Standard Error of the Slope: The standard error of the slope is_____. Because the slope is estimated from the sample, other samples are likely to have differing slopes. The standard error of the slope quantifies the amount of variation in sample slopes that could be expected from different samples.

Recipe for Success: Hypothesis Test for Linear Regression (using data)

1. **Define X & Y in context**

2. **Write your Hypothesis**
   - Null H_0: $\beta_1 = 0$
   - Alternative H_A: $\beta_1 \neq 0$

   H_0: There is no linear relationship between x & y

   H_a: There is a linear relationship between x & y

3. **Define β_1 in context**

   $\frac{\Delta y}{\Delta x}$ = the change in y for every 1 unit change in x

4. **Write the Conditions**
   - Random Sample
   - Linear Scatterplot
   - No pattern in residual plot
   - Distribution of the residuals is normal

5. **Write the Equations**

   $t = \frac{b}{s_b}$ $\quad s_b = \frac{\sqrt{\frac{\sum(y-\hat{y})^2}{n-2}}}{\sqrt{\sum(x-\bar{x})^2}}$

   β_1 = the population slope of the regression line
   t = the number of standard deviations from the mean
   b = the slope of the regression line from the sample
   s_b = the standard error of the regression line
   n = the sample size
   df = n-2

6. **Enter Data**
   - **Stat Edit**
   - Enter X into L_1
   - Enter Y into L_2

7. **Calculate the p-value**
   - **Stat Tests**
   - LinRegTTest→Enter
   - X List- L_1
   - Y List- L_2
   - Freq 1
   - $\beta_1 \neq$ *(or whatever your hypothesis is)*
   - *ReqEQ Alpha Trace*

8. **State the Decision**
   - The p-value is_____
   - If the p-value is less than alpha, Reject the Null
   - If the p-value is greater than alpha, Fail to reject the Null

9. **Write the Conclusion**

   Reject the Null: Our p-value is _____. We reject the Null. There is sufficient evidence at alpha = _____ to suggest that the true population slope is **NOT** zero. There is a linear relationship between_____ and _____.
   Restate the definition of x *Restate the definition of y.*

   Fail to Reject the Null: Our p-value is _____. We Fail to reject the Null. There is not sufficient evidence at alpha = _____ to suggest that the true population slope is different than zero. No linear relationship exists between_____ and _____.
   Restate the definition of y. *Restate the definition of x*

Recipe for Success: Chi-Square Goodness-of-Fit Hypothesis Test

1. **Write your Hypothesis**
   - **Null** H_0: *The _____data follows a _____ distribution*
   - **Alternative** H_A: *The _____ data does not follow a_____ distribution*
     Key words: *distribution, difference, historical, previous, model, matches*

2. **Write the Conditions**
   1. Random Sample (1-sample with 1-variable)
   2. n < 10% of the population
   3. All expected values > 5
   4. Counted data for observed

3. **Write the Equation**

$$\chi^2 = \frac{(O-E)^2}{E}$$

χ^2 = How far away a distribution is from the expected/Null
O = observed Value-the value from the sample
E = the expected or predicted value.
n = the number of categories

4. **Calculate the Expected Values**
   - Sum the number observations in the sample
   - Multiply the number of observations by the Historical **%**

   Note: *For the Expected value of a **Uniform Distribution**; Divide the sum of the sample observations by the number of categories*

5. **List the Expected Values**

6. **Enter Data**
   - Stat Edit
   - Enter Sample Data in column L_1
   - Enter Expected Data in column L_2
   - 2^{nd} Quit

7. **Plug values into the equation** (1^{st} and last Expected values)

$$\chi^2 = \frac{(O-E)^2}{E} + \ldots + \frac{(O-E)^2}{E}$$

8. **Calculate the χ^2 and the p-value**
   - Stat Tests
   - χ^2 GOF-Test Enter
   - Observed: Press 2^{nd} L_1
   - Expected: Press 2^{nd} L_2
   - **df= n-1** *(df is the degrees of freedom)*

9. **State the Decision**
   - The p-value is_____
   - If the p-value is less than alpha, Reject the Null
   - If the p-value is greater than alpha, Fail to reject the Null

10. **Write the Conclusion**

Reject the Null: Our p-value is _____. We reject the Null. There is sufficient evidence at alpha = ___ to suggest that the is _____.
Restate H_A

Fail to Reject the Null: Our p-value is _____. We Fail to reject the Null. There is not sufficient evidence at alpha = ___ to suggest that is _____.
Restate H_A

Recipe for Success: Chi-Square Test of Independence

1. **Write your Hypothesis**
   - **Null H_0:** *The_____and _____are independent*
     Variables being compared
   - **Alternative H_A:** *The_____and _____are associated and are not independent.*
     Variables being compared

   Key words: ***independence, association, relation***

2. **Write the Conditions**
   1. Random Sample (1-sample with 2-variable)
   2. n < 10% of the population
   3. All expected values > 5
   4. Counted data for observed

3. **Write the Equation**

$$\chi^2 = \frac{(O-E)^2}{E}$$

χ^2 = How far away a distribution is from the expected/Null
O = observed Value-the value from the sample
E = the expected or predicted value.
n = the number of categories

4. **Calculate & List Expected Values**
   - 2nd Matrix→Edit →[A]
   - Enter Matrix size *(row x column) do not include totals.*
   - Enter observed Data
   - 2nd Quit
   - STAT TESTS
   - **C: χ^2-Test**
   - 2nd matrix→Edit ↓ [B] *(these are the expected values)*

5. **Plug values into the equation**
   (1st and last Expected values)

$$\chi^2 = \frac{(O-E)^2}{E} + \ldots + \frac{(O-E)^2}{E}$$

6. **Calculate the χ^2 and the p-value**
   - STAT TESTS
   - **C: χ^2-Test**
   - **df= (r-1)(c-1)** *(r =rows & c=columns)*

7. **State the Decision**
   - The p-value is_____
   - If the p-value is less than alpha, Reject the Null
   - If the p-value is greater than alpha, Fail to reject the Null

8. **Write the Conclusion**

Reject the Null: Our p-value is _____. We reject the Null. There is sufficient evidence at alpha = ____ to suggest that the _____are _____ are associated and are not independent. *Restate Variables*

Fail to Reject the Null: Our p-value is _____. We Fail to reject the Null. There is not sufficient evidence at alpha = ____ to suggest that the _____ and _____ are not independent. *Restate Variables*

Recipe for Success: Chi-Square Test of Homogeneity

1. **Write your Hypothesis**
   - **Null H_0:** *The distribution of* _____ *and* _____ *are the same.*
     (1st sample; 2nd sample)
   - **Alternative H_A:** *The distribution of* _____ *and* _____ *are not the same*
     (1st sample; 2nd sample)

   Key words: ***distribution are the same; same***

2. **Write the Conditions**
   1. Random Sample (2-sample with 1-variable)
   2. n < 10% of the population
   3. All expected values > 5
   4. Counted data for observed

3. **Write the Equation**

   $$\chi^2 = \frac{(O-E)^2}{E}$$

   χ^2 = How far away a distribution is from the expected/Null
   O = observed Value-the value from the sample
   E = the expected or predicted value.
   n = the number of categories

4. **Calculate & List Expected Values**
   - 2nd Matrix→Edit →[A]
   - Enter Matrix size *(row x column) do not include totals.*
   - Enter observed Data
   - 2nd Quit
   - STAT TESTS
   - **C: χ^2-Test**
   - 2nd matrix→Edit ↓ [B] *(these are the expected values)*

5. **Plug values into the equation**
   (1st and last Expected values)

   $$\chi^2 = \frac{(O-E)^2}{E} + \ldots + \frac{(O-E)^2}{E}$$

6. **Calculate the χ^2 and the p-value**
   - STAT TESTS
   - **C: χ^2-Test**
   - **df= (r-1)(c-1)** *(r =rows & c=columns)*

7. **State the Decision**
   - The p-value is_____
   - If the p-value is less than alpha, Reject the Null
   - If the p-value is greater than alpha, Fail to reject the Null

8. **Write the Conclusion**

Reject the Null: Our p-value is _____. We reject the Null. There is sufficient evidence at alpha = ____ to suggest that the distributions of _____ (1st sample) and _____ (2nd sample) are the same.

Fail to Reject the Null: Our p-value is _____. We Fail to reject the Null. There is not sufficient evidence at alpha = ____ to suggest that the distributions of _____ (1st sample) and _____ (2nd sample) are not the same.